Risk Sharing and Quasi-Credit

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Abstract

Recent empirical evidence indicates that rural households in the Third World smooth consumption through reciprocal gifts and informal credit but fail to achieve Pareto efficiency in risk sharing. Extending previous models of informal contracts as repeated games, this paper shows that several often described features of informal risk sharing arrangements can be understood as limitations imposed by their self-enforcing nature. We argue that informal credit between friends and relatives is a hybrid transaction, half-way between market exchange and gift giving, whose purpose is to overcome enforcement problems present in pure income pooling arrangements.
Risk Sharing, Quasi-Credit, and the Enforcement of Informal Contracts

Recent empirical studies indicate that rural communities of the Third World achieve a significant, though imperfect degree of consumption smoothing via risk sharing.² The sharing of risk takes two major forms: consumption credit;³ and assistance in kind, i.e., gifts of food and money, labor support, loans of land free of charge, and the adoption of children.⁴ Both ways of exchanging risk depart from standard credit and insurance contracts. Assistance in kind is not a market transaction since no explicit link is made between what is given and an obligation to repay. On the surface, consumption credit appears to be a market transaction, but between villagers it often takes the form of what Platteau and Abraham (1987) have called 'quasi-credit': debt repayment obligations are customarily renegotiated to reflect shocks affecting lender and borrower; loans are often made without interest; access to loans is based on need; and transactions are personalized (Udry (1990), Platteau and Abraham (1987)). Both assistance in kind and quasi-credit, however, contain an implicit obligations to reciprocate: 'I am willing to help you today because I expect you to help me later' (Posner (1980), Platteau (1991)). Individual risk sharing transactions are embedded in long term relationships. The desire to preserve the relationship appears to be the main motivation behind reciprocity and thus the main enforcement mechanism for informal risk sharing arrangements (IRSA) (e.g., Posner (1980), Platteau (1991)).

IRSAs were first formalized by Kimball (1988) and Coate and Ravallion (1993) (see also Fafchamps (1995)). Both use repeated games to demonstrate that an implicit agreement to share risk can be sustained through repeated interaction and thus that promises to assist others can be self-enforcing. Fafchamps (1992) applies a similar approach to situations with information asymmetries. This paper generalizes previous work in several directions and shows the close relationship between non-market transactions like gift giving, and pseudo-market transactions like quasi-credit.

In section 1, we construct a stylized economy and illustrate how repeated interaction can support risk sharing. In section 2 we expose several of the limitations of IRSAs. In particular we explain why IRSAs are vulnerable to increases in risk or risk aversion. In section 3 we expand the set of strategies to include quasi-credit. We demonstrate that quasi-credit can overcome some of the limitations of pure gift giving, thereby providing an explanation of why credit is a dominant form of risk sharing (Rosenzweig (1988), Lund and Fafchamps (1998)). We note that the external enforcement of credit contracts can help informal risk sharing. Conclusions and implications for future work are presented at the end.

Section 1. Informal Risk Sharing

To focus the attention on risk sharing, we follow Kimball (1988) and Coate and Ravallion (1993) and consider an exchange economy in which output cannot be stored, assets cannot be accumulated, and there is no borrowing and lending from the rest of the world. Gains from trade are thus limited to the sharing of risk within the community. There are $N$ individuals in the economy, each with an exclusive claim over a stream of uncertain income. The vector $(y^1_s, \ldots, y^N_s)$ of individual income realizations depends on
the state of nature \( s \in S \). For simplicity, \( s \) is assumed independent and identically distributed over time.

People derive utility from what they consume \( V_i(c^i_s) \). Risk sharing is Pareto improving: no one is risk loving and some people are risk averse, i.e., \( V'' \leq 0 \) for all \( i \)'s and \( V'' < 0 \) for some \( i \)'s.\(^5\) Pareto efficiency requires that the ratio of agents’ marginal utilities be equalized across states of nature (e.g., Mace (1991), Cochrane (1991), Townsend (1994)):

\[
\frac{V'_i(c^i_s)}{V'_i(c^i_s)} = \frac{V'_j(c^j_s)}{V'_j(c^j_s)} \quad \text{for all } i, j, s, s' \quad (1)
\]

Such allocations can in principle be achieved through conditional transfers \( \pi^i_s \) given (received if negative) by individual \( i \) if state of the world \( s \) is realized, i.e., if \( \pi^i_s = y^i_s - c^i_s \) for all \( i, s \). Transfers must of course sum to zero: \( \sum_{i \in N} \pi^i_s = 0 \) for all \( s \).

By virtue of the first welfare theorem, the perfect competitive equilibrium achieves Pareto efficiency in risk sharing. This equilibrium may, however, remain out of reach if explicit risk sharing contracts are difficult or costly to enforce. In Third World villages in particular, legal enforcement has high costs relative to the size of transfers required for risk sharing, and it is hard for external observers to verify whether a breach of a mutual insurance contract has occurred (Hart and Holmstrom (1987)). As a result, the threat to enforce an explicit, legally binding, risk sharing contract through court action is seldom credible. The situation then resembles a prisoner’s dilemma: agents benefit from cooperation but they are unable to commit not to defect ex post. There is market failure.

\(^5\) In addition, we assume that the support of \( y^i_s \) is bounded so that, by the concavity of utility, \( V_i(\text{Sup}_{j=1}^N y^j_s) \) is bounded as well.
If agents interact over time, however, risk sharing can be supported by an informal, i.e., non legally binding but self-enforcing arrangement. This can be shown formally provided that agents are immobile and infinitely lived, or that their conditional probability of interacting for another period remains constant over time (e.g., Kimball (1988), Coate and Ravallion (1993)). These conditions are approximately met in stable rural or fishing communities of the Third World where social obligations and stigma are inherited within dynastic households. Such communities are also those for which IRSAs have been documented.

The repeated risk sharing game can be formalized as follows. Let $\delta \in (0,1)$ be a common discount factor -- or equivalently the product of a discount factor and a constant probability of continued interaction -- and let $Q$ denote a sequence of action profiles or path of the economy. Define $\omega_i(Q)$ as agent $i$’s discounted expected payoff along that path, i.e.:

$$\omega_i(Q) = \sum_{t=0}^{\infty} \delta^t EV_i(\gamma_{s,t} - \pi_{s,t}^i(Q))$$ (2)

where the $\pi_{s,t}^i(Q)$ refer to actual transfers at the end of each period $t$ dictated by action profiles $Q$. Following Abreu (1988), $N + 1$ strategy profiles are sufficient to span the complete set of equilibrium payoffs of this economy: one cooperative risk sharing strategy profile denoted $Q^0$, and one punishment strategy profile $Q^k$ for each of the $k \in \{1, \ldots, N\}$ agents. Agents play according to $Q^0$ as long as nobody deviates, and switch to $Q^i$ following a defection by agent $i$ either to the initial path $Q^0$ or to any of the punishment paths $Q^k$. Abreu (1988) showed that, provided punishments $Q^k$ are the harshest that can

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6 Since the one-shot game has a unique Nash equilibrium, finite repetition of the game cannot enlarge the set of subgame perfect equilibria to include Pareto improving payoffs (Benoit and Krishna (1985)).
be credibly inflicted on deviant agents, adding other more complex strategies cannot expand the set of equilibrium payoffs -- and thus the extent of risk sharing. Voluntary participation to informal risk sharing implies that agents cannot be maintained below the expected payoff they could guarantee themselves by exiting the risk sharing group. Since autarky is the Nash equilibrium of the one-shot game, autarky payoffs serve as maximum punishments:

$$\omega_k(Q^k) = \frac{EV_k(y^k_s)}{1-\delta} \text{ for } k \in \{1,...,N\}$$  \hspace{1cm} (3)

Along the equilibrium path $Q^0$, net transfers between agents depend not only on their own realized income but also on that of all other agents. This imposes high informational requirements as agents must monitor each other’s income to spot defections (Fafchamps (1992), Ligon (1993)). One would therefore expect IRSAs to be more prevalent among tightly knit communities where information circulates freely, e.g., fishing communities where the catch of the day is commonly observed, and farming communities where yields can be visually estimated by all (Platteau and Baland (1989), Platteau (1991)). In this paper, informational issues are assumed away for the simplicity of exposition.

As is well known, the set of subgame perfect equilibria of a repeated game is very large (Fudenberg and Maskin (1986)). IRSAs are no exception. Kimball (1988) and Coate and Ravallion (1993) get rid of the multiplicity of equilibria by positing that a social planner picks the allocation that maximizes the unweighted sum of individual

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7 This does not mean that permanent exclusion is the only possible form of punishment. The expected discounted payoff of a punished agent cannot fall below his or her expected discounted autarky payoff, otherwise the agent would defect from the punishment path. But punishments can be front-loaded (Abreu (1986)): the payoff of the punished agent can temporarily be brought below the autarky payoff provided that the punishment phase is limited in time.
utilities subject to participation constraints. In practice, the choice of an equilibrium is the outcome of a bargaining process within the risk sharing group. Bargaining power probably depends on the threat point of each member but also on their negotiating skills, ethical considerations, past experience, altruism, and ideology, as well as on the group’s polity. The interplay of these factors makes the outcome hard to predict. There is no reason to believe all bargaining processes converge to the allocation picked by Kimball and Coate and Ravallion’s social planner. Much can nevertheless be said about IRSAs by looking at the set of equilibria itself and studying how its boundary evolve as conditions change. This is the approach adopted in this paper.

The set of equilibrium payoffs are comprised between the Pareto efficient frontier (equation 1) and a set of voluntary participation (VP) constraints which must be satisfied along any equilibrium path after the realization of $s$:

$$V_i(y_{s',t}^i) - V_i(y_{s',t}^i - \pi_{s',t}^i(Q^k)) \leq \delta \omega_i(Q^k) - \delta \omega_i(Q^j) + A_i$$

for all $s' \in S$ and all $k=0, 1, \ldots, N$. Participation constraints require that, for any realization of the state of nature $s'$, the short-run gain from deviation $V_i(y_{s',t}^i) - V_i(y_{s',t}^i - \pi_{s',t}^i(Q^k))$ must be smaller than the discounted long-run gain from cooperation $\delta(\omega_i(Q^k) - \omega_i(Q^j))$. It is clear from equation (4) that VP constraints are never binding when $\pi_{s',t}^i(Q^k) \leq 0$. Voluntary participation is problematic only when $Q^k$ requires an agent to help others.

Many social scientists have argued that reciprocity is often reinforced by an ideology or culture that emphasizes the right to subsistence and the corresponding moral obligation to assist someone in need (e.g., Scott (1976), Keyes (1983), Brocheux (1983), Feeny (1983)). To reflect this view, we have added the non-strategic penalty $A_i \geq 0$ to capture the expected subjective satisfaction agents may derive from ’doing the right
thing'. Alternatively, $A_i$ can represent the guilt people may expect to feel for reneging on their promises. Social sanctions other than exclusion from risk sharing -- e.g., exclusion from other forms of social interaction, hazing -- can also be thought as part of parameter $A_i$.\footnote{Equation (4) seem to require that utility be additively separable between the satisfaction derived from consumption and that from social interaction (e.g., satisfaction to conform, guilt from deviation). This is not correct, however. Starting from a non-separable utility $U_i(c, p)$ defined over consumption $c$ and subjective penalty $p$, $A_i$ can be defined as:}

$$A_i = \frac{E[U_i(y^i, p)] - E[U_i(y^i, 0)]}{1-\delta}$$

with $V_i(y) = U_i(y, 0)$. Since the distribution of autarky consumption is regarded as a given in the rest of this paper, the separability assumption simplifies the presentation without affecting the results.

Imposing the $A_i$ penalty is not a strategic decision: other agents need not coordinate to impose the penalty, and the penalty is incurred whenever an agent deviates, irrespective of the magnitude of the deviation.\footnote{The presence of $A_i$ penalties may, under certain conditions, enlarge the set of subgame perfect equilibria in finitely repeated games so that it (nearly) coincides with that of infinitely repeated games (e.g., Fudenberg and Tirole (1993), chapters 5 and 9). In these cases, the infinite horizon is a useful approximation to the finite horizon case. A detailed treatment of this issue is beyond the scope of this paper, however.} This is obviously a simplifying assumption justified by our desire to focus on strategic interaction.

The emphasis that most traditional Third World cultures puts on solidarity has led some to believe that participants to mutual insurance arrangements are solely motivated by altruistic feelings and ethical principles. This view has been severely criticized by Popkin (1979) and others as much too naive. There is plenty of evidence that self-interest motivates behavior in traditional as well as modern societies (for particularly colorful examples, see, for instance, Poewe’s (1989) account of kinship in Zambia). Ethics and well understood self-interest need not be conflictual, however (Posner (1980)). As equation (4) illustrates, they are largely complementary: VP constraints are indeed easier to satisfy and more efficiency in risk sharing can be achieved when $A_i$ is large, that is, when agents are altruistic or feel guilty for letting others down. This can easily be shown.
formally. Let $\Omega(A)$ be the set of subgame perfect equilibrium payoffs corresponding to a particular value of $A$ for all $i \in N$. Then we have:

**Proposition 1:** Suppose $A^1 \leq A^2$. Then $\Omega(A^1) \subseteq \Omega(A^2)$.

Stated in English, Proposition 1 asserts that the set of risk sharing equilibria is larger when non-strategic subjective penalties are large. Since affection is the primary source of altruism, it implies that risk sharing is expected to be strongest among members of the same family or lineage and among friends and neighbors (e.g., Ben-Porath (1980), Foster and Rosenzweig (1995)). Religious fervor also creates strong bonds between groups of converts and can serve as the basis for much mutual insurance and charity.\footnote{Poewe (1989), for instance, documents how evangelical churches in Zambia have replaced moribund traditional solidarity networks. Geertz, Geertz and Rosen (1979) and Cohen (1969) give examples of the role of Muslim sects in enforcing cooperation among traders.} Furthermore, the altruistic desire to help others and the ability to feel guilty for failing to do so can be cultivated through education and enhanced through personal interaction (Platteau (1994a, 1994b)). For all these reasons, it is not surprising that Third World communities often describe IRSAs in emotional or moralistic terms. Altruism alone, however, may be insufficient to support much risk sharing. In the remainder of this paper we seek to understand when altruism and ethics are most put to the test by studying the extent to which self-interest can, on its own, support risk sharing. Participation constraints impose limits on the degree of risk sharing and Pareto efficiency that can be achieved.

**Section 2. Gifts and Risk Sharing**

Let us first focus on 'stationary' strategies, that is, to strategies that depend only upon the current state of nature $s$, not on past transfers. Strategies are affected by the past history of play only inasmuch as defection and punishments are concerned. These
strategies resemble what we might call gift giving in that what is given today is not function of what was given yesterday. In the case stationary strategies, VP constraints take a simpler form:

\[
V_i(y_s^t - y_s^{t'} - \pi_s^{t'}) - V_i(y_s^{t'} - y_s^t) \leq \frac{\delta}{1-\delta} E[V_i(y_s^{t'} - y_s^{t}) - V_i(y_s^t)] + A_i
\]  

(5)

VP constraints (5) impose serious restrictions on IRSAs that can account for a number of stylized facts about gifts and risk sharing that cannot easily be accounted for in models exclusively based on altruism (e.g., Becker (1981), Ravallion and Dearden (1988)). First of all, self-interested risk sharing cannot be supported when agents are impatient or when they do not expect to interact for long, that is, when \( \delta \) is low. This is a well known property of repeated games (e.g., Fudenberg and Maskin (1986)). An implication is that risk sharing is unlikely among highly mobile populations, such as urban migrants (see, for instance, Hart (1988) for evidence in Ghana). The shape of the equilibrium payoff set for various values of \( \delta \) is shown in Figure 1. The figure illustrates the well known result that the set of equilibria of a repeated games shrinks as agents get more impatient.

By pooling the resources of agents with different income streams, risk sharing can in principle redistribute incomes from agents with high average incomes to those with a low average income. Such redistribution is achieved by granting to the poor a larger share of the welfare gains from risk sharing. In this case, solidarity not only reduces temporary poverty; it also palliates chronic poverty. Self-interest, however, puts limits on

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11 The anthropological literature has nevertheless emphasized that the magnitude of gifts often depends on what was given in the past, so that gift giving is seldom memoryless in those societies that resort to it the most. This important caveat should be kept in mind when applying this model to actual gift giving practices.

12 Figure 1 was constructed by computer simulation, using stationary strategies and assuming that \( A = 0, N = 2 \) and \( V_i(y) = \log(y) \)
Proposition 2: As $\delta$ decreases (impatience increases), expected utility gains from risk sharing must be shared more equally.

Proposition 2, which is illustrated in Figure 1, implies that a redistribution of welfare is harder to achieve in communities with a short time horizon because all participants, rich and poor, insist on receiving an equal share of the welfare gain from mutual insurance. This is a general result that does not depend on the shape of the utility function or the distribution of risk. What could mitigate this result is if subjective penalties $A_i$ are an increasing function of expected utility. In this case, 'rich' yet impatient participants could still be incited to accept redistributive equilibria. Efforts to instil a sense of equity and fairness in the mind of participants -- thereby raising the subjective penalty they incur for refusing to share -- might thus be seen as an attempt to manipulate $A_i$ so as to temper the selfishness of the rich.

Proposition 2 characterizes the tension between insurance and redistribution that is inherent to informal safety nets in terms of expected utility, but it does not say anything about realized transfers. To these we now turn. It has been observed that IRSAs occasionally 'break down' during famines in the sense that people most in need fail to receive assistance. Sen (1981), for instance, notes that during the Ethiopian famine of 1974 many domestic servants were laid off by their employer even though it was clear that they would starve. Greenough (1982), pp.215-225 cites examples of nuclear households that break up during famines. The absence of risk sharing in bad times is difficult to reconcile with altruism but it can be explained by self-interest considerations. As Coate and

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Ravallion (1993) have shown, the discount factor required to induce an agent to share risk goes up as the income of others goes down: the lower the income of others, the harder it is to ensure voluntary income pooling. If the limited contribution that a relatively fortunate agent is willing to make is insufficient to keep others from severe hardship, then the IRSA will appear to break down.

Efficiency gains from risk sharing increase with risk and with aversion toward risk (e.g., Arrow (1971)). One would therefore expect risk sharing to be more prominent when incomes are more variables and agents are more risk averse. Kimball (1988) and Coate and Ravallion (1993) indeed present simulation results in which risk sharing increases with risk and aversion toward risk. It is, however, possible to come up with examples in which the reverse is true, as illustrated in the following proposition:

**Proposition 3:** Take any stationary IRSA in which some efficiency gains from risk sharing are realized (A1). Suppose that there exist at least one binding participation constraint for one agent, say agent $i$. Let $N$ be the number of participants to the IRSA and let $S$ be the number of possible states of the world. Then:

1. Provided certain technical assumptions are satisfied (see proof), there exists a concave transformation of the utility function of agent $i$ such that the participation constraint is violated.
2. If $S \geq 2N + 2$, there exists a mean preserving spread in risk such that agent $i$’s participation constraint is violated.

The first part of the proposition states that, provided certain technical conditions are satisfied, it is always possible to make agents more risk averse in a manner that makes any risk sharing equilibrium unsustainable. The second part asserts that it is always pos-
sible to find a mean-preserving increase in risk that makes any risk sharing equilibrium unsustainable. An immediate corollary of Proposition 3 is that we can never prove a general proposition stating that an increase in risk or in risk aversion raises risk sharing, although this may be true in particular cases such as the simulations reported by Kimball (1988) and Coate and Ravallion (1993).

Behind Proposition 3 is the realization that two opposite forces are at work in any IRSA: an increase in risk or risk aversion raises the gains from risk sharing, thereby raising the right hand side of the VP constraints (5). But it may also increase the subjective cost of sharing risk \( V_i(y_{s',t} - y_{s',t} - \pi_{s',t}(Q^0)) \), particularly if helping others entails one’s immediate starvation.\(^{14}\) Depending on the net effect of these two forces, the set of sustainable equilibria may shrink or expand. Proposition 3 thus helps explain why risk sharing often appears limited or inexistent among the extremely poor and the destitute. It also suggests that a reduction in average group income and an increase in income variability, e.g., because of increased population pressure or of environmental degradation, may undermine an existing IRSA.

**Section 3. Risk Sharing and Credit**

In practice, gifts and transfers seldom are the dominant form of consumption smoothing in the Third World; consumption credit is typically a more important avenue for sharing risk within the community (e.g., Rosenzweig (1988), Townsend (1995), Lund and Fafchamps (1998), Alamgir (1980), p.156-157). The resemblance between such consumption credit and market transactions is, however, largely superficial. The amounts transacted often are too small to justify court action; contracts must be self-enforcing.\(^ {15}\)

\(^{14}\) Unless starvation is already certain, in which case sharing does not make any difference.

\(^{15}\) In that, consumption credit formally resembles the way sovereign debt contracts have been modelled
Moreover, as recent evidence has shown, consumption credit is often implicitly combined with some form of insurance: debts can be forgiven, repayments can be postponed, and actual contractual performance typically depends on the lender’s and the borrower’s situation at the time of repayment (e.g., Udry (1990, 1994), Platteau and Abraham (1987)). We now show that quasi-credit can be understood as a non-stationary strategy equilibrium of a long term, implicit risk sharing arrangement. Our treatment of the issue differs from previous work by Kocherlakota (1996) and Ligon, Thomas and Worrall (1996) in that we do not impose a principal-agent structure to our model and focus instead on equilibrium sets.¹⁶ Our results can be regarded as a generalization of the works of these authors.

Non-stationary strategies

So far we have focused on stationary strategies, that is, on strategies in which transfers depend only on the current state of nature. These strategies alone can support cooperation but less restricted strategies can achieve more. We investigate a special class of non-stationary strategies, one in which agents are individually rewarded for contributing to the group. As it turns out, this class of strategies establishes a formal link between risk sharing and credit practices.

Formally, consider strategies in which the consumption of agent \( i \) is the sum of three terms: realized income \( y_{i,s} \), net transfers \( \pi_{i,s} \), and a reward \( w_{i,t} \) that, for the time being, we

¹⁶ A principal-agent structure would not be restrictive if it was only imposed in the first period of the game. By varying the agent’s reservation utility, one could simply span the whole boundary of the equilibrium set. Imposing the restriction in each period, however, forces an additional structure upon the model, structure that Kocherlakota (1996) and Ligon, Thomas and Worrall (1996) exploit to derive a precise characterization of the dynamic path of payoffs.
shall call brownie points:

\[ c^i_{s,t} = y^i_{s,t} - \pi^i_{s,t} + w^i_t \]  

(6)

Brownie points \( w^i_t \) can be thought of as the net wealth or goodwill capital of individual \( i \).\(^{17}\) They are not function of the current state of nature.

Let \( y^i_{s,t} \) and \( w^i_t \) be the vectors of individual incomes and brownie points, respectively. Transfers \( \pi^i_{s,t} \) map from the cross-product of realized incomes and brownie points at time \( t \) into the real line, i.e.:

\[ \pi^i_{s,t} = \Pi^i(y^i_{s,t}, w^i_t) \]  

(7)

Transfers depend on past history through \( w^i_t \). Brownie points reward positive transfers to others, but since transfers are themselves function of realized incomes and brownie points, we can write the law of motion of \( w^i_t \) in the following reduced form:

\[ w^i_t = W^i(y^i_{s,t-1}, w^i_{t-1}) \]  

(8)

Brownie points are normalized so that \( \sum_{i \in N} w^i_t = 0 \) for all \( t \).

With these new assumptions, participation constraints can be rewritten:

\[ V_i(y^i_{s,t-1}) - V_i(c^i_{s,t-1}) \leq \sum_{u=1}^{\infty} \delta^u EV_i(c^i_{s,t-1}) - \frac{\delta}{1-\delta} EV_i(y^i_{s,t-1}) + A_i \]  

(9)

In this new notation, stationary strategies correspond to restricted transfer functions in which \( \Pi(y, w^0) = \Pi(y, w^1) \) for all \( y, w^0, \) and \( w^1 \). Since non-stationary strategies are less restrictive than as pure gift giving, they should allow more risk sharing. This intuition is confirmed by the following proposition, which generalizes results obtained by Kocherlakota (1996) and Ligon, Thomas and Worrall (1996):

\(^{17}\) \( w^i_t \) need not be expressed in monetary terms. In the hau system of exchange discussed by Sahlins (1972), for instance, goodwill takes a purely symbolical form.
**Proposition 4:** Let $A$ be the set of perfect equilibria supported by stationary strategies and let $B$ be the set of perfect equilibria supported by non-stationary strategies. Then $A \subseteq B$.

Restated in english, Proposition 4 says that more risk sharing can be supported with non-stationary strategies. Perhaps the best way to understand what is behind Proposition 4 is to provide an example. Suppose there are two agents with the following concave utility function:

<table>
<thead>
<tr>
<th>Consumption</th>
<th>1</th>
<th>1.5</th>
<th>2</th>
<th>2.5</th>
<th>3</th>
<th>3.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Utility</td>
<td>-2</td>
<td>1</td>
<td>2</td>
<td>2.5</td>
<td>2.75</td>
<td>3</td>
</tr>
</tbody>
</table>

There are three equally likely states of the world, numbered 1 to 3. Corresponding income vectors of agents 1 and 2 are (1, 2), (2, 1), and (3, 3). Altruism is ignored, i.e., $A_1 = A_2 = 0$. We compare two risk sharing schemes. The first one, which is stationary, stipulates transfers from agent 1 to agent 2 of -0.5, 0.5, and 0 in states of the world 1, 2, and 3, respectively. The second scheme is like the first in states of the world 1 and 2. But in state 3, it stipulates that an agent receives a payment of 0.5 if the previous state of the world was 1 or 2 and he or she gave a transfer to the other agent. This payment can be thought of as contingent credit repayment. We show that the second scheme can be supported for a lower discount factor than the first.

Consider the first scheme. Given the symmetry of the game, there is only one participation constraint to consider, that when one of the agents must give 0.5 to the other. Expected utilities from cooperation and autarky are, respectively:

\[
EV_i(c^i_{s,t}) = \frac{1}{3}V(3) + \frac{2}{3}V(1.5)
\]  
\[
EV_i(y^i_s) = \frac{1}{3}V(3) + \frac{1}{3}V(2) + \frac{1}{3}V(1)
\]
The difference between the two is $2/3$. The gain from defection is $V(2) - V(1.5) = 1$. The VP constraint is satisfied for $\delta \geq 0.6$.

Now consider the second scheme. There are two participation constraints to satisfy. The first one, as in the stationary scheme, ensures that payment is made in states of the world 1 and 2. The expected utility from autarky and the gain from defection are unchanged, but the expected utility from cooperation has increased because of the reward in state 3. It is now:

$$\delta \left[ \frac{1}{3} V(3.5) + \frac{2}{3} V(1.5) \right] + \frac{\delta^2}{1-\delta} \frac{2}{3} V(1.5) + \frac{1}{6} V(3.5) + \frac{1}{6} V(2.5)$$

The minimum $\delta$ at which the participation constraint is satisfied is now 0.588. The second participation constraint makes sure that payment of 0.5 in state 3 is self-enforcing. The minimum $\delta$ at which voluntary payment is made is 0.364. The second scheme can thus supports risk sharing in bad times even when $\delta < 0.6$. More general examples can be found in Ligon, Thomas and Worrall (1996).

**Quasi-credit**

As the above example suggests, quasi-credit belongs to the class of non-stationary strategies. To see why, let us split $\pi^t_{s,t}$ into two parts: a loan $l$ and a pure transfer $\tau$. Let the interest rate be $r$. Then:

**Proposition 5:** For any interest rate $r$, and any function $W^t(y, w)$ and $\Pi^t(y, w)$, there exist a loan function $l^t(y, w)$ and pure transfer function $\tau^t(y, w)$ such that:

$$c^t_{s,t} = y^t_{s,t} - l^t(y^t_{s,t}, w_t) + w^t_l - \tau^t(y^t_{s,t}, w_t)$$

$$w^t_l = -(1+r)l^t(y^t_{s,t-1}, w_{t-1})$$

Proposition 5 states that any non-stationary equilibrium can be mapped into -- and thus represented by -- a quasi-credit equilibrium. It establishes the formal resemblance
between quasi-credit and a non-stationary strategy equilibrium of a repeated risk sharing game. Quasi-credit can thus be considered as a form of insurance. Proposition 4 also teaches us that, in the absence of enforcement problems, efficient risk sharing could be achieved through gifts alone. It is enforcement problems that are the reason for the existence of quasi-credit: by establishing a direct link between what agents give today and what they expect to receive tomorrow, quasi-credit rewards giving over and above what pure gift giving can achieve. As a result, it is able to overcome some of the limitations imposed by participation constraints and thus raise efficiency (see also Ligon, Thomas and Worrall (1996)).

Many of the features of quasi-credit that have been noted by observers of rural Third World societies (e.g., Scott (1976), Popkin (1979), Platteau (1991), Basu (1986), Gluckman (1955)) are puzzling when quasi-credit is looked at as a regular market transaction. Treating quasi-credit as the equilibrium of a repeated risk sharing game helps explain many of them. First, there is no sense in which the interest rate on quasi-credit contracts clears the market: as Proposition 5 demonstrates, the interest rate is indeterminate. This helps explain why so many Third World consumption loans between equals carry no explicit interest (e.g., Townsend (1995), Lund (1996)) and why interest rates sometimes vary wildly between transactions in the same village and time period (e.g., Udry (1990)). Second, loan repayment is conditional on subsequent shocks; default and postponement are anticipated and implicitly accepted beforehand. This stands in contrast with regular credit contracts which are expected to be repaid in most if not all circumstances. Third, access to credit is the means by which mutual insurance is organized. Loans are therefore rationed: in order to get a loan, one must show sufficient need. Quasi-credit at zero interest rate is not meant to be used for investment purposes.18

18 Townsend (1995) provides evidence that investment loans in Thai villages do carry an interest and are treated differently from consumption loans. Lund (1996) provides similar evidence for the Philippines.
Fourth, because repayment is only guaranteed by continued participation to the IRSA, quasi-credit loans are unlikely to be made to someone whose expected future contribution to or gain from risk sharing is low. Transactions are thus not anonymous but entirely personalized: that \( i \) got a loan from \( j \) does not mean that \( k \) can get a loan from \( j \). All these predictions are in agreement with the characteristics of risk sharing networks described in Lund (1996) and Lund and Fafchamps (1998).

**Debt Forgiveness and Debt Rescheduling**

Many of the transfers that appear in Proposition 5 serve to offset credit obligations. Does it matter how these transfers take place? As Proposition 6 demonstrates, the answer is yes: more efficiency in risk sharing can be achieved if debt can be postponed and not simply forgiven. Let \( A \) and \( B \) be as in Proposition 5. Let \( C \) stand for the set of subgame perfect equilibria that can be achieved when debt cannot be rescheduled, that is, when \( W(y, w_0) = W(y, w_1) \) for all \( y, w_0 \) and \( w_1 \) (A6). It follows that:

**Proposition 6:** \( A \subseteq C \subseteq B \).

Stated in English, Proposition 6 simply establishes that more risk sharing can be achieved if debt postponement is allowed. In the example presented in the previous subsection, debt could not be postponed. It is easy to expand this example to show that debt rescheduling increases the reward for giving and thus reduces the discount rate required for the participation constraint to be satisfied, in which case we would have \( C \subseteq B \) as well. This is left as an exercise for the reader. Proposition 6 thus provides a possible explanation for why debt repayments often are postponed and rolled over instead of being simply forgiven (e.g., Udry (1990), Lund (1996)).
Formal credit

Although many consumption credit transactions remain informal, some, like loans from money-lenders for instance, are somewhat more formal and often include a credible threat to seek external enforcement. Since the respect of debt repayment obligations is more easily verifiable by an outside party than the state of nature on which transfers $\pi_s$ depend, there are good reasons to suspect credit contracts to be enforceable even when mutual insurance obligations are not. We now show that the possibility of external enforcement, even if imperfect, helps risk sharing. The reason is that the effectiveness of quasi-credit is limited by the requirement that credit obligations $w^j_i$ be self-enforcing: the more debt an agent accumulates, the more tempting it is for him or her to defect from the IRSA. These limitations are reduced if credit contracts can be externally enforced.

Assume that a credible external enforcement technology exists for credit contracts. If a debtor defaults on a loan, penalties can be inflicted. Let the expected discounted utility cost of these penalties be an non-decreasing function $P(w^j_i)$. Let us also assume that penalties are finite, i.e., that $\lim_{w \to \infty} P(w) = \overline{P} < \infty$. The participation constraint for risk sharing can then be rewritten:

$$V_i(y^j_i) - V_i(c^j_i) \leq \sum_{u=1}^{\infty} \delta^u E[V(c^j_{x,u})] - \frac{\delta}{1-\delta} EV_i(y^j_i) - P(w^j_i) + A_i$$

(15)

The only difference with equation (9) is $P(w^j_i)$: the use of credit contracts inflicts heavier penalties on agents who defect on their risk sharing obligations. Let $D$ be the set of perfect equilibria supported by externally enforceable credit contracts as defined in equation (15). Since harsher punishments support more risk sharing, we get:

**Proposition 7:** $B \subseteq D$. 
Stated differently, Proposition 7 asserts that more risk sharing can be achieved if implicit risk sharing arrangements are combined with explicit and externally enforceable debt contracts. External enforcement can thus only improve efficiency.\textsuperscript{19} Pure credit alone would not achieve the same result. Furthermore, external enforcement helps only if if the penalty for default \(P(w)\) is finite. If contractual default is never allowed, individuals never borrow more than the annuity value of their minimum income. If their minimum possible income is zero, they never become net borrowers, however small the probability of a zero income is (e.g., Zeldes (1989), Carroll (1992), Fafchamps (1996)).\textsuperscript{20} In this case, consumption smoothing can only be achieved through pure gift giving. Zame (1993) demonstrates that even contingent contracts cannot, in general, be efficient unless penalties for default are not too high, that is, unless contract repudiation is tolerated in certain circumstances. Allowing debt obligations not to be met in certain cases makes credit contracts resemble quasi-credit: they \textit{de facto} mix credit with insurance. Proposition 7 complements these earlier results by showing that insurance can in turn be made more efficient by externally imposing penalties for the non-respect of credit contracts.

Proposition 7 opens a large gray area between non-market transactions -- the income pooling arrangements discussed in section 1 -- and pure market transactions -- contracts that are enforced exclusively through \(P(w_t)\). Most real world transactions probably stand somewhere in between. There often is an implicit arrangement between parties to renegotiate the terms of the explicit contract, either by forgiving part of the debt or

\begin{footnotesize}
\begin{itemize}
\item[\textsuperscript{19}] Although it may increase inequality in the long run; see Fafchamps (1998c).
\item[\textsuperscript{20}] The only exception is when an agent’s utility without the loan is already \(-\infty\). Desperation can lead agents to borrow under threats of extreme punishment if they do not repay.
\end{itemize}
\end{footnotesize}
by postponing contractual compliance (e.g., Kranton (1996), Fafchamps (1996, 1997), Bigsten et al., (1998), Fafchamps and Minten (1998)). In this gray area, transactions share some of the characteristics of quasi-credit: rationing according to need; a certain indeterminacy in interest rates, solved either through individual variation in interest rate or by fixation around a focal point; *de facto* conditional loan repayment; and personalized, repeated transactions. They also may display characteristic that are associated with well functioning markets: limits on the individual variation of interest rates; mobility between sources of finance; free access to credit within a certain range. Although contract enforcement issues naturally raise a host of information asymmetries problem, Proposition 7 suggests that enforcement alone can account for many observed features of credit markets (e.g., Stiglitz and Weiss (1981)) even in the presence of perfect information (see Fafchamps (1998a, 1998b) for illustrations).

**Conclusion**

The theory of repeated games is often criticized for generating a multiplicity of equilibria. Other efforts at modeling IRSA using repeated game theory have felt obliged to impose additional requirements that select one equilibria out of many (e.g., Kimball (1988), Coate and Ravallion (1993)). We have shown here that numerous insights can be gained by considering the set itself.

Recent work has indicated that rural communities of the Third World fail to achieve perfect risk sharing (Townsend (1994, 1995), Morduch (1991), Udry (1994)). The theory presented here suggests a possible explanation for it, namely, that voluntary participation constraints limit the extent of mutual insurance. It also generates testable hypotheses regarding the relationship between risk sharing, altruism and time horizon (e.g., Foster...
and Rosenzweig (1995)); risk sharing, risk and risk aversion (e.g., Ligon, Thomas and Worrall (1996)); and interest rates on quasi-credit. Formally testing these implications is left for further research.

In this paper we assumed away asset accumulation. Yet, Third World communities achieve a fair degree of consumption smoothing not only through the pooling of risk but also through the accumulation of assets like gold or livestock (e.g., Rosenzweig and Wolpin (1993)). Villagers may also be able to borrow from banks, moneylenders or traders (e.g., Udry (1990, 1994)). Allowing individual asset accumulation enables the community to protect itself against collective shocks. But it reduces the gain from mutual risk sharing and may weaken IRSAs (e.g., Ligon, Thomas and Worrall (1996)). Other manifestation of power, such as debt peonage and labor bonding, can also be studied within the framework proposed here. These issues are left for future work.

21 The accumulation of assets changes our model only inasmuch as these assets can be exchanged against consumption goods with the outside world. The accumulation of a good, real or symbolical, that circulates exclusively within the community behaves like brownie points in our model (see section 2).
Appendix: Proofs of All Propositions

Proof of Proposition 1: Each point on the boundary of the equilibrium set can be found by maximizing one agent’s expected utility subject to satisfying all participation constraints and maintaining other agents at a given expected utility level. By varying the expected utility of other agents and by repeating the process for all agents, we can span the whole boundary of the equilibrium set. Now, participation constraints with \( A^1 \) are a restricted version of participation constraints with \( A^2 \). Therefore, by Le Chatelier principle, the maximum expected utility with \( A^2 \) lies weakly above what can be achieved with \( A^2 \) for all expected utility levels of other agents. Every point on the boundary of \( \Omega(A^1) \) thus lies weakly below every point on the boundary of \( \Omega(A^2) \). This proves the proposition. Strict inclusion occurs whenever \( \delta \) and \( A^1 \) are low enough for some participation constraints to be binding. A strictly higher \( A^2 \) then is sure to release the binding participation constraints somewhat and to strictly enlarge the set of equilibria.\[\Box\]

Proof of Proposition 2: For any given IRSA \( \pi^i_s \), the right hand side of equation (4) decreases with \( \delta \). Consider an arbitrary agent \( i \) and state of nature \( s \). Let \( \delta \) fall to the point where equation (4) is binding for that \( i \) and \( s \). In order to further decrease \( \delta \) while still ensuring payment of \( \pi^i_s \), agent \( i \) has to be compensated in other states of nature \( s' \). Compensation by other agents is possible as long as some of their participation constraints are not binding. A lower \( \delta \) thus forces agents who contributed little and whose participation constraints were therefore less binding, to contribute more. If \( \delta \) drops further, eventually no agent is left without binding participation constraint.\[\Box\]

Proof of Proposition 3: We drop \( i \) subscripts for simplicity of exposition.

Part 1: (Case 1) If \( V_i(y) > -\infty \) for all \( y > -\infty \), assume that there exist a binding participa-
tion constraint for that agent at which he or she does not get his or her highest possible income (A2). (Case 2) If \( \lim_{y \to y^*} V_i(y) = -\infty \) for \( y^* > -\infty \), assume that agent \( i \) prefers the probability of a \( -\infty \) utility tomorrow than a \( -\infty \) utility today (A3). If utility is undefined below a particular value of \( y \), set it equal to \( -\infty \). Let \( \tilde{y} \) denote the realized income of agent \( i \) at the binding participation constraint. Let \( X \) stand for the right hand side of the participation constraint \( V(\tilde{y}) - V(\tilde{y} - \tilde{\pi}) \). Since the constraint is binding and there is risk sharing (A1), the right hand side of the participation constraint is strictly positive and \( X > 0 \). Construct a concave transformation of \( V(y) \) as follows: for all \( y \geq \tilde{y} \), leave \( V(y) \) unchanged; for all \( y < \tilde{y} \), rotate agent \( i \)'s utility by a factor \( k > 1 \), i.e., \( i \)'s utility becomes \( kV(y) - (k-1)V(\tilde{y}) \).

Case 1: With this new utility function, the right hand side of the participation constraint becomes \( kX \): the utility loss of complying with IRSA obligations has been stretched by a factor \( k \). The right hand side of the participation constraint after the transformation of \( V(y) \) can be decomposed into three parts:

\[
(k-1)V(\tilde{y})[Pr(y \leq \tilde{y}) - Pr(y-\pi \leq \tilde{y})] + \int_{\tilde{y}}^{\tilde{y}-\pi} [V(y-\pi) - V(y)]ds + \int_{\tilde{y}}^{\tilde{y}} [V(y-\pi) - V(y)]ds
\]

Let the three terms of the above sum be denoted \( A, kB, \) and \( C \). The right hand side of the participation constraint before the transformation was simply \( B + C \). By (A2), \( C \neq 0 \). The participation constraint after the transformation is violated iff \( A + kB + C < kB + kC = kX \), that is, iff \( k > A/C - 1 \). Since \( A \) and \( C \) are constants that do not depend on \( k \), such a \( k \) always exists.

Case 2: If \( \lim_{y \to y^*} V(y) = -\infty \) for \( y^* > -\infty \), then as one increases \( k \), agent \( i \)'s utility may fall to \( -\infty \) for income realizations in the support of \( y_s \), or for possible consumption realizations \( y_s - \pi_s \). When this happens \( i \)'s expected utility falls to \( -\infty \) and the construction that
we used in case 1 no longer works. It is still possible, however, to pick a $k$ large enough that the utility agent $i$ derives from contributing $\tilde{\pi}$ is $-\infty$. Then, by (A3), the participation constraint is violated. □

**Part 2:** Let $\omega_s$ stand for the probability that the realized state of the world is $s \in \{1, 2, \ldots, S\}$. By assumption, $1 > \omega > 0$ for all $s$. Let $s^\prime$ be the state of the world when the participation constraint is binding and let $Z_i$ be the value of the binding participation constraint $V(y_{s^\prime}) - V(y_{s^\prime} - \pi_{s^\prime}^i)$. Introduce the following notation:

$$A_s^i \equiv V(y_s - \pi_s) - V(y_s)$$

$$\mu^j = \frac{1}{S} \sum_{s=1}^{S} \omega_s y_s^j$$

$$\sigma^j = \frac{1}{S} \sum_{s=1}^{S} \omega_s (y_s^j - \mu^j)^2$$

Furthermore, let $\Omega$ be the vector of probability weights $(\omega_1, \omega_2, \ldots, \omega_S)$, $B_i$ be the vector $(B_1^i, \ldots, B_S^i)$, $\Phi$ be the vector of income means $(\mu^1, \ldots, \mu^N)$, $\Sigma$ be the vector of income variances $(\sigma^1, \ldots, \sigma^N)$, $1$ be a vector of ones, and $\Xi$ and $\Psi$ stand for the $S \times N$ matrices of incomes and squared deviations from income mean.

We know that

$$\Omega^\prime B^i = Z^i$$

$$\Omega^\prime \Xi = \Phi^\prime$$

$$\Omega^\prime \Psi = \Sigma^\prime$$

$$\Omega^\prime 1 = 1$$

We want to show that it is possible to find another set of probability weights $\hat{\omega}_s$ such that each agent faces the same expected income, the variance of each agent’s individual income has increased, and agent $i$’s participation constraint is violated. Formally we want to find a vector $\hat{\Omega}$ such that

$$\hat{\Omega}^\prime B^i = Z^i + \varepsilon$$
where $\varepsilon > 0$ is a scalar and $\Gamma$ is a vector of strictly positive numbers. The above can be rewritten:

$$
\hat{\Omega}^i[z \Psi \Sigma \Phi \Gamma 1] = [Z^i + \varepsilon, \Phi', \Sigma' + \Gamma', 1]
$$

(A1)

Let $\rho$ be the rank of the matrix $[B^i \Xi \Psi]$. Clearly, $\rho$ cannot exceed $2N + 2$. Thus if $S \geq 2N + 2$, there exists at least one set of probability weights (several if the inequality is strict) such that equation (5) is satisfied for any arbitrary $\varepsilon$ and $\Gamma$. Since by assumption the initial probability weights $\omega_s$ are all strictly positive, the linearity of equation (A1) implies that there exists a set of numbers $\varepsilon$ and $\Gamma$ such that equation (A1) is satisfied and $\hat{\omega}_s \geq 0. \Box$

**Proof of Proposition 4:** Similar to that of Proposition 1.1. \Box

**Proof of Proposition 5:** Set

$$
l^i(y_{s,t}, w_t) = -1/(1+r)W_i(y_{s,t}, w_t)
$$

$$
p^i(y_{s,t}, w_t) = \pi^i(y_{s,t}, w_t) - l^i(y_{s,t}, w_t) \Box
$$

**Proof of Proposition 6:** Apply Le Chatelier principle as in Proposition 1.1. \Box

**Proof of Proposition 7:** Apply Le Chatelier principle as in proposition 1. \Box
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Figure 1. Risk-sharing and impatience

\[ \delta = 0.95 \]
\[ \delta = 0.85 \]
\[ \delta = 0.73 \]
\[ \delta = 0.65 \]