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Alexei Birulin

ON MONETISING A BARTER ECONOMY

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В работе рассматривается возможность перехода от бартера к денежному рынку как переход между системами с двумя типами информации. В качестве бартера выступает приватное "знание" торгового партнера, рынок же интерпретируется как информационная сеть. В работе представлено несколько моделей, основанных на работе Kranton: "Частный обмен: самоподдерживающаяся система", однако в дипломной работе рассматриваются как процессы распада существующих бартерных связей, так и создания новых. Одна из представленных моделей учитывает экстерналии несовершенного рынка, в другой рассматриваются только парные взаимодействия. Вводя в рассмотрение издержки, постоянно действующие в процессе бартерного обмена, мы рассматриваем возможность монетизации экономики с так называемым "вынужденным" бартером. В модели проявляются множественные равновесия, в том числе и такие, в которых приватная бартерная связь доминирует рыночную сеть по Парето. Кроме того рассмотрена модель с "делимыми деньгами", которая характеризуется равновесием, в котором все расчеты осуществляются по единой цене. Эта модель может служить для обоснования результатов, полученных в моделях с "неделимыми деньгами". Работа завершается микроэкономической моделью с двумя последовательными аукционами, которая показывает важность учета не только размеров, но и адресатов эмиссии и указывает на возможность "цепных реакций" в монетизации.

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This paper deals with the issues of institutional forms interactions, namely with the problem of transformation of barter economy into monetary market. Several models originated from R.Kranton: «Reciprocal Exchange: Self-Sustaining System» are presented in the paper. Barter formation is studied in random matching framework with indivisible money. Both the models with pair wise interactions and accounting for network effects are discussed. With introduction of the "digestion" cost in barter transactions the possibility of monetising the economy, where the good obtained via barter is inferior to that bought at the monetary market. The analysis suggests the multiplicity of equilibrium with some, where monetary market remains inferior to private barter links. In chapter II the model with divisible money is presented and the existence of stationary equilibrium characterised by prevailing single-price in transactions is shown. This chapter may be treated as one giving support to the results of the models with indivisible money. Finally a microeconomic model with two subsequent auctions is presented. This model suggests the significance of the correct choice of the addressees of the emission and possibility of "chain effects" in monetising. The thesis is finished with a discussion of models and policy suggestions.

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Contents.

ON BARTER ECONOMY MONETIZING

Introduction

Rapid increase of barter is one of the most remarkable phenomena in Russian economy during the recent years. Enormous size of barter transactions (undergoing market reforms and highly developed technology make it even incredible) nowadays is casual for Russian enterprises. According to Russian Economic Barometer data Aykytsionek [1] not only the unprecedented increase of barter (from 6% of total operations in 1992 to 41% in 1997) is apparent, but there is no evidence of barterization slowing down. Among the other reasons of using barter the respondents of REB survey mentioned liquidity constraints -47%, intention to facilitate the exchange - 39%, high transaction costs of monetary transactions -17%. From the products obtained through barter channels 2/3 is used in production, 1/10 - for consumption, 1/7 - for market (monetary) sales, 1/10 - for re-barter. All barter according to Aukutsionek [1] could be divided into "voluntary" barter when the goods that anyway would be bought from the same producers are bartered. The share of this barter is accounted for 60% of the total size of barter. The remaining 40% are - "inferior" barter.

The existing literature on barter mostly concentrates on various aspects of asymmetric information problem and its implications and endogeniging of transaction costs. Williamson, Wright [12] and Kiyotaki, Wright [9] develop a model of production and exchange with uncertainty concerning the quality of commodities and study the role of fiat money. Without private information only high-quality goods are produced, while with private information there can be equilibria (sometimes multiple) where low-quality commodities are produced and money can increase welfare. In efficient monetary equilibria agents adopt strategies that increase the probability of acquiring high-quality output.

Banerjee and Maskin [2] study a barter economy in which each good is produced in two qualities and no trader can distinguish between the qualities of two goods he neither produces nor consumes. It is shown that in competitive equilibrium there exists a unique good (for one the discrepancy between qualities is smallest) that serves as medium of exchange. This good acts as an intermediate in every trade. Equilibrium is inefficient because production of the medium good would be lower if it were not for its mediating role. Introducing fiat money enhances the trade by eliminating the distortion. However, high inflation drives traders back to the commodity medium.

Prendergast and Stole [13] examine the issue of asymmetry of information about the quality of good, allowing for opportunistic behavior by buyers and sellers. Fiat money efficiently enables trade on contractible goods (with observable at the moment quality). Dynamic reciprocity (implicit contracts enforced through reputation concerns) are aided by the presence of money through voluntary monetary payments which relax the inter-temporal incentive compatibility requirements of the relationship. It is also shown that money could as well affect the willingness of agents to provide non-contractible goods (future contracts) because it reduces the sanction from failing to fulfill the obligations and possibly induce inefficient pricing and production decisions.

Ellingsen presents in [3] the model with private information about liquidity constraints. The paper goes on to show that when the seller is uncertain about the buyer's financial position, countertrade can serve as a screening device. In equilibrium seller finds it optimal to offer a menu of contracts such that buyers who are not liquidity constrained choose to pay exclusively in cash, whereas buyers that are constrained prove their hardship by making part of the payment in kind.

In Prendergast and Stole [14] the issue of hidden actions inside the firm is examined. It is shown that non-monetary exchange could reduce inefficiency via effecting the allocation of rents, as agents respond strategically to the existence of these rents and second, non-monetary trade improves the ability of imposing sanctions on agents who act dishonestly.

Kranton [11] examines the interaction between personalized, long-term reciprocal exchange relationship and anonymous market exchange. The analysis shows that the benefits from the reciprocal exchange can actually derive from the prevalence of the reciprocal itself. So the model exhibits "thin market" externalities that lead to hysteresis: whether reciprocal or market relationship will prevail depends on the initial distribution of population. Thus inefficient outcomes are possible.

Our studies were stimulated with the paper by Kranton [11]. The environment was changed considerably. Firstly, we introduced the possibility of both barter pair decomposition and fusion into reciprocal. Secondly, we allow not only agents with money, but also agents without it meet with other producers and offer barter link at their discretion. Total market size and the amount of barter in our model are endogenous variables, while the number of buyers is treated as an exogenous parameter equal to money supply.

The paper contains several models and is organized as follows. In section I the models with indivisible money are presented. In our models market operations involve state dependent transaction costs, so that barter appear to be the first best for all the agents. This corresponds to the voluntary barter in Russian economic terminology. We examine barter formation in two different settings. The first includes network effects that agents face at the anonymous market. Namely, because of coordination imperfections the sellers are not insured against inefficient frictions when a few of them are interacting with a single counterpart. The second model with different timing deals only with pairwise interactions. Namely, it allows for the optimal spreading of agents in the economy eliminating the network effect. Introducing "digestion" cost in barter transactions we then study the possibility of monetizing the economy, where the good obtained via barter is by definition inferior to this bought at the monetary market. The analysis suggests the multiplicity of equilibria with some, where monetary market remains inferior to private barter links.

Section II is devoted to the model where barter formation is studied in pairwise interactions framework with divisible money. This analysis was stimulated by the paper of Green and Zhou [5] where the version of Kiyotaki and Wright economy with an equilibrium with all transactions occur at a single-price is described. In our paper the environment of Green and Zhou was suited for studying barter - monetary market interactions. Very preliminary model including barter and characterized by stationary equilibria with prevailing of single-price in transactions is presented in this thesis. The results of the model presented in section II can be extended to the analysis of barter formation in the economy with digestion costs. Thus section II may be treated as one giving support to the results of the models with indivisible money.

In section III a microeconomic model with two sequential auctions is presented. This section is not directly connected to the preceding chapters and can be treated as a separate study. The analysis was stimulated by the idea about price discrimination driven barter, see Guriev, Kvasov [8]. Namely, because of difference in money holdings the richer agents may impose an externality on the poorer allowing bidding up the prices. The poorer agents then choose barter as the only the alternative means of payments available. The model with two sequentialuent auctions allows for studying the agent response on monetizing. The analysis suggests the significance of the proper choice of the addressees of the emission and possibility of "chain effects" in monetizing. Namely, properly directed money injection may cause the overall decrease in prices and “activate” the money holdings of the agents who before the injection used to barter the goods.

The closing section is devoted to the discussion of all the models and policy suggestions derived from these models.

Section I. **Indivisible money**.

I.1**. *Environment: Production Technologies and Preferences***.

In this section we discuss the trade-off between barter and market exchange not as a trade-off between two fashions of exchange: one consumption good for another or consumption good for fiat (with no intrinsic value) money; but rather than the trade-off between two types of information. Bilateral barter link corresponds to private information, whereas market trade corresponds to network system. Under our usual assumptions about market, holding money means, firstly, that the holder knows where to buy; secondly, is insured that if he has «enough» money he will get the good; and thirdly, he knows what to produce in order to get the money. While the market becomes «thin» because of very low payable demand (because of very few buyers) the third assumption is violated, while the first two become even more rigid.

The basic features of the economy structure are similar to *Kranton’s.* [11]

1. The whole range of goods and services in the economy consists of infinite number of consumption commodities plus a good that we call fiat money. Consumption goods are non-storable, whereas the fiat money is the good that has the following properties: costless storability, no transaction costs (no wear and tear except possible inflation, zero volume in the pocket), infinite costs of fabrication.
2. For each agent the consumption of *m* randomly selected commodities from the whole range gives the equal level of utility *U*. Thus consumption bundle of the representative consumers consists of *m* goods, consumption of the good from the bundle brings a utility of *U,* while out of the bundle - 0. The consumption bundle for each agent assumed to be predetermined and fixed, but is his private information. In other words for each and every consumer there exists a probability $x \in (0,1)$ (equal for all consumers) that randomly selected good is from his consumption bundle. For each good there are *m* consumers that prefer this good, or randomly selected consumer prefers this good with probability *x*.
3. The economy is populated with a continuum of infinitely living agents each with a given discount rate. The total population is normalized to one. Time proceeds in discreet periods. Each period an agent can both produce a unit of a given indivisible good and consume a unit of good form his consumption bundle. One period is defined as the time it would take for an agent to produce (one half-period), obtain a good from another agent and consume (another half-period). Let δ be the discount factor per half-period.
4. Each agent is maximizing the discounted expected utility $V_t = E_t\left(\sum_{s=t}^{\infty} \delta^{s-t} Y\right)$,
5. Here *Y* equals *U* - the utility from one act consumption if there is consumption at a given date and equals zero if the agent has not managed to consume

anything that date. So the above functional includes the expectation of the total discounted consumption path estimated on the information available at time t.

6. Production and consumption can take place at the free (involving fiat money exchange) market or in bilateral reciprocal exchange. For each commodity there is an infinite number of potential producers. We consider production at no cost, but this assumption is not restricting one. Production takes place at particular locations, in order to produce an agent should be at the production place where the given commodity is produced. The number of production places for free market participants is normalized to L, the location of these production places is common knowledge. Only one producer can occupy each production place.
7. It is also assumed that producers can search for the empty place at no cost and no time, but consumers cannot observe the "distribution" of producers and consumers. So there is a theoretical possibility that the agent come to the empty production place or that more than one agent will come and claim for one unit of good. Finally, it is assumed that agents cannot consume their own output so that there are always benefits to trade over self-sufficiency.
8. Individuals can be engaged in lasting reciprocal (without fiat money) exchange. The production for this exchange takes place at private production place, the number of these places is infinite and their location is private information of each barter pair. Reciprocal exchange is sustainable in case of double coincidence of wants condition satisfied. Under our assumptions about agent preferences this happens with probability of x^2. (Each agent desires the product of his partner with probability x). Thus we study so-called voluntary barter (the terminology by *Aukutsionek* [1]). The share of this type of barter is estimated to be approximately 60% of the total size of barter transactions. [1]

9. Memory is assumed to be short term. Agents remember only the very last barter partner and only while the link is maintained.

We have made the following extensions to *Kranton's* model.

1. Bilateral reciprocal exchange once started continues the next period with probability of P^2. This probability in our analyses is treated as exogenous and independent on the duration of the existing barter exchange and independent on the existing market conditions. The agents are as well allowed to terminate the link if they wish at no extra punishment.
2. Not only agents with money, but also agents without it can meet with other producers. The latter as well as the former can offer barter link if they wish.

The agents are operating in a virtual production city with production places. All agents are assumed to play mixed strategy splitting into «settlers» currently occupying production place and «nomads» travelling between production places. «Settled seller» cannot display his presence and can not more than sit in the production site where the product of his type is supposed to be produced. This means that everybody knows where big shoes are produced, but nobody knows which shoemaker is working at the moment. Nomad seller (agent without money who in fact is searching for a barter partner) can visit only one production place during one period of time.

I.2. Interactions.

I.2.a. *Seller-Seller Interactions with Network Effects.*

Once the agents meet and the action starts they reveal what they are producing and decide whether they can barter. If at one place appear a few agents appropriate for barter, the «host» of the place is assumed to choose a partner from them at random. Note also that an agent has the following trade-off. Waiting for the partners he is insured against competitors sitting at the same place, but he is not sure that somebody will come to the production place he is currently occupying. At the same time an agent while travelling can get to the empty production place, or not be selected from the group of competitors.

Note that because of perfect symmetry of tastes telling the truth about who is producing what is weakly dominant strategy. What is important is that only one part in each trade group knows exactly what is produced by the counterpart, so the agents can only guess about what is needed for the barter exchange.

Lemma 1. Nash equilibrium rule of splitting into «settles» and «nomads» is 50/50 for sellers.

Proof. A seller is optimizing the probabilities of getting into barter, which because of symmetry of tastes is similar to simple maximization of probability of meeting with the other sellers.

Let the seller play the mixed strategy with probability μ of being «nomad» and $1-\mu$ of being «settled».

Then being «settled» he faces the following probability of making barter. This is the probability that at least one «nomad» comes to his production place and will appear to be appropriate for barter

$$B_d^R = 1 - \sum_{n=0}^{x\mu s} (1-x)^n \binom{x\mu s}{n} \cdot \left(\frac{1}{xL}\right)^n \left(1 - \frac{1}{xL}\right)^{x\mu s - n}$$
$$= 1 - \left(\frac{1}{xL}\right)^{x\mu s} \sum_{n=0}^{x\mu s} \binom{x\mu s}{n} (1-x)^n (xL-1)^{x\mu s - n}$$

There are xL similar production places, so $1/xL$ is the probability that some agent chooses the given spot. $(1-x)$ - is the probability that this agent will be inappropriate for barter.

The above formulae could be transformed into $B_d^R = 1 - \left(1 - \frac{1}{L}\right)^{x\mu s}$

In the asymptotic approximation with $L \to \infty$ the probability to find a barter partner could be rewritten as $B_d^R = 1 - e^{-x\mu s}$ where with s the fraction of sellers in total population is denoted. In the appendix is shown that for the agent currently «nomad» the probability of getting into barter is

$$B_u^R = \frac{1-\mu}{\mu}(1 - e^{-x\mu s}).$$

An agent is optimizing his behavior equating the probabilities

$$1 - e^{-x\mu s} = \frac{1-\mu}{\mu}(1 - e^{-x\mu s})$$

The above has a solution of μ=0.5 Suppose all other sellers are playing the strategy of spitting 50/50 and one decides to deviate. Suppose he sees "nomad" option better and then deviates increasing μ. This leads to the decrease of "nomad" option in the above equation and because of the assumed initial parity "nomad part" appears to be less attractive than "settled". So the deviation was not reasonable. Q.E.D.

I.2.b. Buyer-Seller Interactions.

A buyer - agent with money can also choose between traveling and sitting. While traveling he is insured against coming to the empty production place and against other buyers-competitors coming to the same production place. We assume this “insurance” to be an intrinsic feature of money additional to the above. Digression: that seems not to be a big problem when money is divisible, two buyers can compete for the good if they wish, bidding up the price, but with indivisible money they have too little discretion in making their offers. When two agents offer the same money, but one of them does not get the good, what this money is? We assume, maybe too optimistically, "indivisible hand of market" to be powerful enough to solve the problem of coordination described above. Note also that the assumption that not more than one buyer appears at one trading place is not very binding when the number of sellers (people without money) greatly exceeds the number of buyers (people with money). This assumption is plausible as long as the number of buyers is less than half of the total population size. We will consider only the environments satisfying this assertion.

A buyer while settled can shout: "I have money, I need big shoes!!!" so that sellers can hear him. For a seller is weakly better to transact with a buyer (they can make a barter, but if they cannot, the seller can at least raise money) than with another seller. Then a buyer is insured against nobody coming to his place al least for monetary transaction. In this environment we partly endogenize the idea about "indivisible hand" capturing the postulate that everything can be bought for money, but allow market to stay "thin" for barter traders.

All that is formalized in the:

Assertion. Money holding and monetary trade ensures an immediate access to the good.

Note that the assertion in fact consists of two parts. Firstly, the agent with money should get an access to the good and secondly, the seller should accept the money. Money is accepted in trade as a realization of agent expectations. Namely: seller is willing to exchange the good for the “pieces of paper” because he believes that tomorrow the seller, he will want to transact with, will also accept these “pieces” in exchange for the good. We will construct the environment so that the assertion above will be satisfied as a part of equilibrium trading strategy of the agents.

The first part of the assertion can be implemented by introducing the time of inspection of the good offered for barter (that for fiat money is treated as zero). Then the agents, who are willing to trade in money, will have a possibility to travel before the inspection of goods is started. They spread in the economy so that there is eventually not more than one agent with money at one trading place. Note that time costly inspection also means that the seller via bargaining with that agent can treat as an outside option only ex ante not ex interim valuation of the probability of getting the good via barter. (They cannot observe whether there is somebody ready to barter or not in the queue.) We will later show that the seller will then trade in money though barter is more beneficial, but not certain.

Note that under the assertion above a buyer is indifferent between settled or nomad states when he is trading in money (he will get the good for sure in either case).

Clearly with our assumptions about bargaining protocol of buyer-seller interaction we can distort the optimal splitting rule for sellers (see lemma 1) and thus decrease the rate of barter growth. Taking this effect into consideration is too difficult and maybe not reasonable. Note that the agents are spreading so that «normal» trading environment is one «settler» facing one «nomad» and all the other terms of interactions are outliers though quite possible.

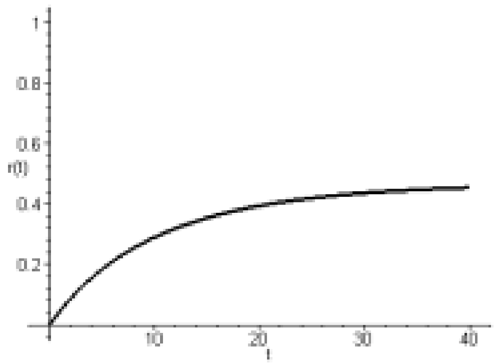
When buyers can commit not to use money in the transactions the story is simple: all treat buyers as sellers without money and do not pay heed to their privileges. This is not more than the change of the number of sellers without money. When buyers can commit to use only money that also is not a big problem, they are indifferent between nomad and seller states and so we may assume their 50/50 splitting. Sellers response on that is clearly 50/50 splitting.

The difficulty arises when buyers cannot commit to use only money because of dynamic inconsistency described later. There are equilibria where buyers use their money only as a passport for an easier access to the good, but they wish not to transact via money. Then the buyer can choose a barter partner among a few agents being settled. At the same time he is not more than vis-a-vis with the seller when he is nomad himself. Let us assume that there is a mechanism of fair spreading of nomad sellers among settled buyers. Then the probability to meet with a buyer for a settled seller is b/s (the number of nomad buyers to the nomad sellers). Then when buyers split 50/50 into nomads and settlers (they do not have strong reasons to choose another rule as they are indifferent) the same probability b/s of meeting with buyer is assigned with a nomad seller. All the other interactions do not affect the equilibrium 50/50 splitting rule by Lemma 1.

Further we denote with b and s the numbers of buyers and sellers not engaged in the barter links at the moment respectively. With R_b we denote the number of agents in barter pairs in which there is one agent with money. With R_s we denote the number of pairs with no money holder. Note that if there were no money in the economy and the only possibility of trade were forming of barter links the number of these links would evolve accordingly to:

$$\frac{\partial R_s}{\partial t} = (1 - e^{-xs})s - 2(1 - P^2)R_s$$

with $s + R = 1$ and appropriate initial condition.



The first term in the above equation corresponds to barter formation, the second to barter fission because of some exogenous reasons. This equation has an exponential solution with additional dampening due to the network effect. The more there are sellers still searching for barter, the easier is to find the barter partner. Correspondence with some features (saturation in approaching 40%) in the empirical data could be suggestive. See, for example: *Guriev, Ickes, Rybakov*, (1999) [6].

Finally we state that buyer-buyer interaction never happens because each buyer is weakly better off choosing a seller.

I.3. Trade Protocols and Behavior Analysis.

Finding a barter partner means engagement into a lasting barter link. Thus each agent is assigned two indices: concerning the occupation and money holdings. Agents currently *engaged* in barter are further denoted with *R*, agents *not engaged* - with blank index. Agents having money - being *buyers* are further noted with *b*, not having it *sellers* are further noted with *s*. Hereafter $V_R(b)$ and $V_R(s)$ stand for the values of the expectation operators of future consumption for the agents, currently engaged in a barter link, and having money or not having it correspondingly. $V(b)$ and $V(s)$ stand for these values for the market agents (without current private link). We consider stationary and state-dependent environments, so these values under a given number of sellers and buyers are not more than four points in two-dimensional space. Engagement into barter link, acts of buying and selling imply not only consumption but also strategic changes in the expected future «income».

The *V* values are related with some obvious inequalities

$$V(b) \geq V(s)\,,\ V_R(b) \geq V_R(s) \qquad (1a)$$

If money matters, to have money is better than not to have.

$$V_R(b) \geq V(b)\,,\ V_R(s) \geq V(s) \qquad (1b)$$

Barter link implies minimum one act of exchange, and forming this link each agent does not more than postpones his market relationships, becoming not worth off. So it is always optimal to engage into barter if it is possible. Since

$$V(b) \geq U + \delta V(s), \tag{2a}$$

From the fact that being a buyer implies one act consumption and then becoming a seller and (1b) follows:

$$V_R(b) \geq U + \delta V(s) \tag{2b}$$

Then a buyer always prefers to renegotiate into barter if a seller accepts this renegotiation.

The key assumption for the following analysis is:

$$U + \delta V(s) \geq x\delta V_R(b) + (1-x)\delta V(b) \tag{3}$$

Suppose a buyer has made his offer of barter (has named the good he can produce). If the good is appropriate for barter a seller will accept the barter because of (1b). If the seller rejects the barter the buyer has an option to buy the good for money. The corresponding expected value of his «future» - left handside of (3). In contrast he can wait till next round, then offer a barter and get it with probability x. The corresponding expected value of the «future» - right handside of (3). Note that being rejected with barter the buyer finds himself in the worst possible situation. If he does not transact in money now, (if (3) is violated) he will newer offer it in the future.

Lemma 2. When $U + \delta V(s) \geq x\delta V_R(b) + (1-x)\delta V(b)$ is satisfied an agent with money (buyer) never mimic the behavior of the agent without money.

Proof. An agent without money has the expected value of future: $\Pi\delta \cdot V_R(b) + (1-\Pi)\delta \cdot V(b)$, where Π is the product of probabilities: Prob1) that his good is appropriate for barter at the given production place. Prob2) that the agent is chosen among the others ready to barter at this place. Prob3) that there is no agents with money at this place. Since Π and x are strictly positive probabilities; $\Pi \le x$ and '=' only if there is only one agent present; $V_R(b) \ge V(b)$. Then

$\Pi\delta V_R(b) + (1-\Pi)\delta V(b) \le x\delta V_R(b) + (1-x)\delta V(b) \le U + \delta V(s)$, because of (3).

The very right handside is agent's with money «prudent strategy» outcome, so that he is guaranteed this level of «future». Q.E.D.

Let us look more closely at the inequality (3). When it is satisfied the agent can show his money to all competitors and thus ensure that he will obtain the good either via money or via barter. Then all the competitors may give up competition at this production place and travel to another. Let us examine seller's behavior. When received take-it-or-leave-it offer from the agent with money the seller accepts barter if it is appropriate (that is his best option anyway) and accepts money if the good is not appropriate for barter. When (3) is violated and inequalities:

$$\Pi\delta V_R(b) + (1-\Pi)\delta V(b) \le U + \delta V(s) \tag{3a}$$

$$U + \delta V(s) \le x\delta V_R(b) + (1-x)\delta V(b) \tag{3b}$$

are satisfied the buyer prefers using money as a «passport» for easier access to the good, but never exchanges it for the good. But all the other competitors know about this as well as the seller. There is the following inconsistency in (3a, 3b): all the competitors treat the agent with money as a casual barter agent, while the agent with money demands for some distinguished position (he wants to bargain first). We can indeed treat this as a dynamic inconsistency: when (3a) is satisfied the agent claims that he will pay in money before the trade starts. But the agent cannot credibly commit to this, as it is not optimal for him to hold the claim after the trade starts. Note that we are discussing not the situation when agents arranged to trade in money but then discovered that they can simply barter. The situation is that they cannot barter, but the agent with money is not going to pay in money but rather wants to travel to the other production site.

Seller's «rejection of his application for a distinguished status» cannot serve as a source of the commitment for the money holder. Once the seller announces about this «rejection...», the agent claims to pay in money (he CAN commit now), but when the action starts seller's «veto» has no more power, and the money holder CANNOT commit to pay in money. To solve this dynamic inconsistency problem we should introduce some contact with a third party (for example raising fines for inconsistent behavior) or credit history which may help because agents are infinitely alive or may not help because of the discount.

To sum up, definitely there are some regions where market network is «too weak» comparing to the private information link. There are some doubts about whether the authorities should back the money, which does not actually act as a medium of exchange. Let us concentrate further on the parameters region where (3) holds, so money does matter as a medium of exchange.

Note that when (3) holds the buyer always get the good in the production place he is currently at, either via barter or via money if seller rejects barter.

The seller then has an option to choose between renegotiating into barter and accepting only money, and renegotiates into barter when:

$$V_R(s) \geq V(b) = xV_R(b) + (1-x)(U + \delta V(s)), \tag{4}$$

where

$$V_b = \frac{x\alpha + (1-x)U}{1-x\beta} + \frac{1-x}{1-x\beta}\delta V(s),$$

Under $\beta \leq \delta$ and $U \leq \alpha$ inequality (4) always holds so the seller is always better off becoming barter agent without money then market agent with money.

Then all the people without money can get the good at a given spot only if there are no agents with money at this production place.

We finish this part with the obvious claim that it is never optimal to postpone trade waiting for better conditions in the stationary environment.

To sum up, *money is inferior to barter once a partner is found, but is better than barter when the agent still faces searching cost*. Reference to *V.Polterovitch* [15] should be made here. Drastic form of transaction cost decrease (possibly considerable while searching for the partner and falling to zero when the partner is found) clearly is the acute form of costs of using barter decrease due to learning and coordination effects described in *V.Polterovitch* [15].

Depending on the exogenous parameters of the environment and the ratio b/s there are the following parameter regions:

Region I: (3) is satisfied, seller always prefers barter to money and money acts as a medium of exchange.

Region II: inequality (3a) is violated and money has no power.

Region III: (3a) and (3b) are satisfied- dynamic inconsistency in buyer behavior.

We restrict attention to region I and assume that under changes in b/s the system is still in region I.

In region I a seller gets into barter either with a buyer or (when there is no buyer) with another seller with an appropriate good. A seller gets the money when a buyer cannot barter with her. A seller remains seller when nobody came to his production place. The value of seller's occupation in the form of the possible future states with the transaction probabilities:

$$V(s) = \left[x \frac{b}{s} + \left(1 - \frac{b}{s}\right)\cdot(1 - e^{-xs})\right]\cdot V_R(s) + (1-x)\frac{b}{s}\delta V(b) + \left(1 - \frac{b}{s}\right)\cdot e^{-xs}\delta V(s)$$

A buyer has a priority in getting into barter and is always paying in money when he can not barter. His value of the "future":

$$V(b) = xV_R(b) + (1-x)(U + \delta V(s))$$

$V_R(\cdot) = U + P^2 \delta V_R(\cdot) + (1-P^2)\delta V(\cdot)$, with P^2 the probability of barter lasting till next period is denoted. Then: $V_R(b) = \alpha + \beta \cdot V(b)$, $V_R(s) = \alpha + \beta \cdot V(s)$, where

$$\alpha = \frac{U}{1-\delta P^2}, \beta = \frac{\delta(1-P^2)}{1-\delta P^2}$$

The population dynamics in region I is described with the following system:

$$\frac{\partial R_b}{\partial t} = 2xb - 2(1-P^2)R_b$$

$$\frac{\partial R_s}{\partial t} = \left(1 - \frac{b}{s}\right) \cdot (1 - e^{-xs/2})s - 2(1-P^2)R_s$$

$$\frac{\partial s}{\partial t} = (1-P^2)(R_s + \tfrac{1}{2}R_b) - \left(1 - \frac{b}{s}\right) \cdot (1 - e^{-xs})s - xb$$

$$\frac{\partial b}{\partial t} = (1-P^2)R_b - xb$$

With appropriate initial conditions.

This system is characterized with converging to some level of barter solution. (The corresponding figures see below in the section "Pairwise Interactions".)

Spreading money in this economy (turning some sellers into buyers via government expenditures, for example) we improve efficiency, facilitating the transactions that otherwise were not conducted (some people could have not transacted because they had no money and had no barter partners). But what is peculiar with this setting is that injecting money we simultaneously increase the rate of barter formation and the final “size of barter”. This happens because money eliminates «frictions» concerning with inefficient competition of several sellers at one production place. Due to this "frictions" buyer-seller links form more rapidly than seller-seller ones. Monetary injection though suppressing seller-seller links formation, facilitates process of buyer-seller links creation.

This is the right place to discuss whether the barter described in the model is the one that we want to straggle with. Direct reciprocal exchange, barter by definition, in our setting is efficient (superior to any other form of trade) once the appropriate partner is found (we discuss a voluntary barter). Let us suppose that everybody has met his trading partner and is maintaining the link. This is the most efficient economic environment we can imagine. There is no sense in introducing any money then, even if the money provides an access to some ("perfect" or "thick") *anonymous* market with no need for private information, which is the essence of the *private* link. Trade-off between anonymous market and private link actually is the trade-off between two types of *information*.

Reference to *Kocherlakota's* "Money as Memory" [10] could be made here. Kocherlakota's main result is some equivalence between money holdings (when observable) and "trading history" of the person. We should note that the environment in "Money as Memory" is considerably different from matching one we use, though Kocherlakota shows that the framework of "Money as Memory" encompasses random matching world. The bargaining protocol we use is similar to "Money as Memory" (accounting for indivisibility of money). Applying money-memory equivalence we may consider the agent's trade in money as (in some sense) the confirmation of obeying the rules of fair market play. Then the result of more rapid formation of barter links in "monetized" economy is the consequence of the "increased credibility" provided by money injection.

I.4. Pairwise Interactions.

To put aside “parasitic” effects of money on barter creation discussed above let us change the environment in the following fashion. Agents can be engaged in lasting barter and belong to the "free market". All free market agents have enough time to travel and spread optimally in the economy if they wish before the inspection of the good starts. That will lead to the pure vis-a-vis interactions. One agent with money faces one agent without it. Or two agents without money are trying to form a barter pair. Then some sellers are randomly selected to serve buyers and the others split 50/50 into "settlers" and "nomads" and interact with each other. One part in each pair is endowed with information about what is produced by the counterpart, or equivalently the counterpart makes his type observable. Note that the agent does not benefit from obtaining this information because of perfect symmetry of tastes. Actually it makes no difference between knowing where to go for a trade and sitting in a production place (as one is insured that some agent will come and try to bargain with him). This leads to the fact that any couple may barter with probability x.

In that environment money clearly acts only as a possible medium of exchange. There are $b+\frac{s-b}{2}=\frac{b+s}{2}$ trading couples at the moment. With s we denote the number of "market" sellers that are not engaged in barter links at the moment. A representative "market" seller is chosen to interact with a buyer with probability $\frac{b}{s}$ and with another seller with probability $\frac{s-b}{s}$. Then the seller:

can appear in a *barter link* with probability x

become a *buyer* with probability $\frac{b(1-x)}{s}$

remain the *seller* with probability $\frac{(s-b)(1-x)}{s}$

A buyer:

can appear in *barter* with probability x

become a seller with probability (1-x)

Then values assigned with current occupations:

$$\left.\begin{aligned} V(s) &= xV_R(s) + \tfrac{(1-x)b}{s}\delta V(b) + \tfrac{(1-x)(s-b)}{s}\delta V(s) \\ V(b) &= xV_R(b) + (1-x)(U + \delta V(s)) \end{aligned}\right\} \qquad (5)$$

$V_R(b) = \alpha + \beta \cdot V(b)$, $V_R(s) = \alpha + \beta \cdot V(s)$, where α, β as above.

Barter is first best option both for buyers and sellers and the population dynamics is described with:

$$\frac{\partial R_b}{\partial t} = 2xb - 2(1-P^2)R_b$$
$$\frac{\partial R_s}{\partial t} = (s-b)x - 2(1-P^2)R_s$$
$$\frac{\partial s}{\partial t} = (1-P^2)(2R_s + R_b) - (s-b)x - xb$$
$$\frac{\partial b}{\partial t} = (1-P^2)R_b - xb$$

with initial conditions. Note that in this "linear" framework additional money has no externality on the rate of barter formation described above. "Limited" monetary injection (turning some sellers into buyers) improves the efficiency in the economy simultaneously increasing the rate of barter formation. Note that we have eliminated "frictions" in seller-seller interactions and hence it is naturally to observe more rapid barter formation. See the figure next page.

I.5. Barter with Searching and Digestion Cost.

Commonly and especially often in Russian literature barter is considered as by definition *inefficient* trade inferior to monetary one. The cause of the existence of this trade is often treated as a simple deficit of money. This is suspicious in the framework with divisible money and we will come back to this issue later, but is natural when money is indivisible. To study the effects concerning with the inefficient barter trade we change the environment introducing some additional permanently acting cost into barter transaction.

Consider the framework of the preceding section with pairwise interactions with additional:

*Assumption***.** The good obtained via barter is always inferior to that bought with money.

In environment with a digestion cost getting the money may become the top-priority goal rather then searching for an appropriate barter partner. But even though for the sellers that have not got money at the moment is still optimal to form the private barter link for one shot of the game and then again try to find a buyer with money. Note that if they both do not succeed in finding money it will be optimal to join each other again for one more shot, etc. Memory is supposed to be short term, people remember only the very last barter partner and only if they have not transacted with money after they terminated the link. We may treat this as a breach punishment that each of the partners impose on the other. Then the agents face the following *trade-off: memory about private partner versus better good in the market.*

We would like to point out that only seller-seller pairs now form barter links.

Lemma. In stationary environment the buyer - agent with money never offers barter transaction.

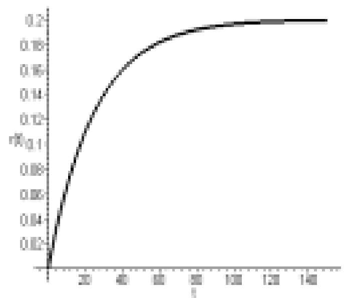
Proof. Let suppose he does. Then he will use his money when barter relationship ends. But with nontrivial discount that is not optimal: he should rather consume a luxury good today and inferior one tomorrow. Q.E.D.

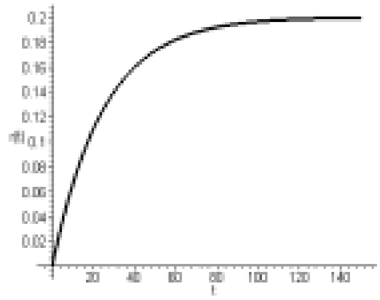
Digression: the above lemma seems to be the consequence of money indivisibility. When the agent anticipates an increase in "money demand" he may

With network effect with spreading before the action

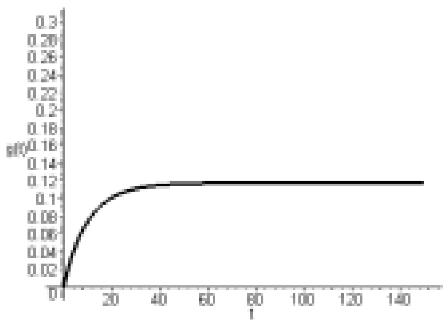
initial conditions b=.2, s=.8

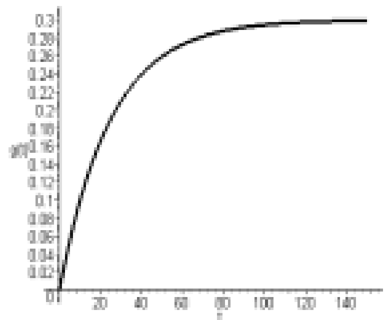
buyer – seller links





seller-seller links





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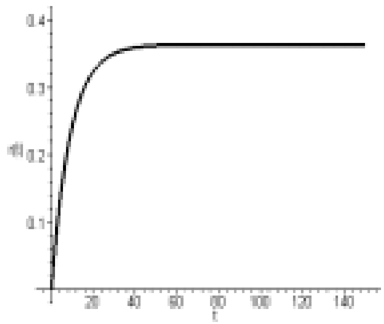
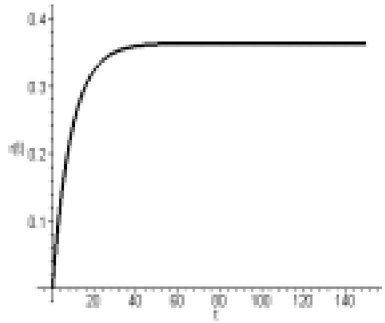
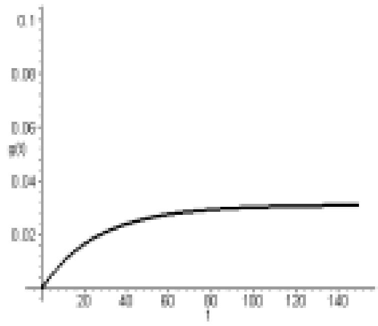
initial conditions b=.4, s=.6

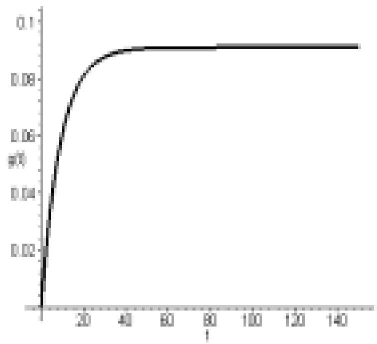
buyer-seller links





seller-seller links





prefer to engage in barter now to save money and use it when price of money in terms of good will go up.

Let suppose memory "overweighs", agents do not terminate their links. Then they observe that the system converges into barter city, free market depletes and the ratio of b/s becomes more favorable, closer to "thick" market value b/s=1. Remember, under our assumptions market may be "thin" only for sellers, buyer always can find the good if he is paying in money. Remember also that we denote with *s* the number of sellers not currently having barter partners. Then the system inevitably match some cut-off value of b/s after which the *good* "overweighs", the agents are actively seeking for the buyer with money and breach a barter contract once they meet buyer. This leads to drastic change in "goods supply", now all the sellers enter free market (at least for one day) and offer their good. The ratio of b/s drops immediately such that "memory" again becomes more attractive. Some of the agents will then renew their barter links, the other may found new ones, but considerable part of agents will loose the "memory" and the system will appear far from the "rebounding" value b/s.

The key point here is how do agents form their expectations. We assume the agents to be rational enough to understand the existence of the “rebound effect”. Then they do not accept any money unless they are not sure that money stock is increased so much that market is thick enough for all the agents alive. More formally this means that the system appears in the region where the ratio of total money stock to the total sellers population ensures:

$\frac{\Delta U}{1-\delta} < V_R(s)$, where $V_R(s)$ - the value of barter link. For the setting where breach (in order to get the money) is optimal it is presented below.

Barter is terminated when somebody in the currently existing pair finds an agent with money. Both agents from pair find buyers with probability $\left(\frac{b}{s}\right)^2$, only one finds with probability $\frac{2b}{s}\frac{s-b}{s}$. Barter lasts next period with probability $\left(\frac{s-b}{s}\right)^2$.

$(s-R)$ free market sellers find buyers with probability $\frac{b}{s}$, engage in barter with probability $x\frac{s-b}{s}$ and otherwise remain free sellers. Buyers with money definitely exchange it for good becoming free sellers. Note that in the above formulas b and s coincide with total money stock and total number of agents without money in the economy.

Then the occupation values for free market agents:

$$V(s) = x\tfrac{s-b}{s}V_R(s) + \tfrac{b}{s}\delta V(b) + \tfrac{(1-x)(s-b)}{s}\delta V(s)$$

$$V(b) = U + \delta V(s).$$

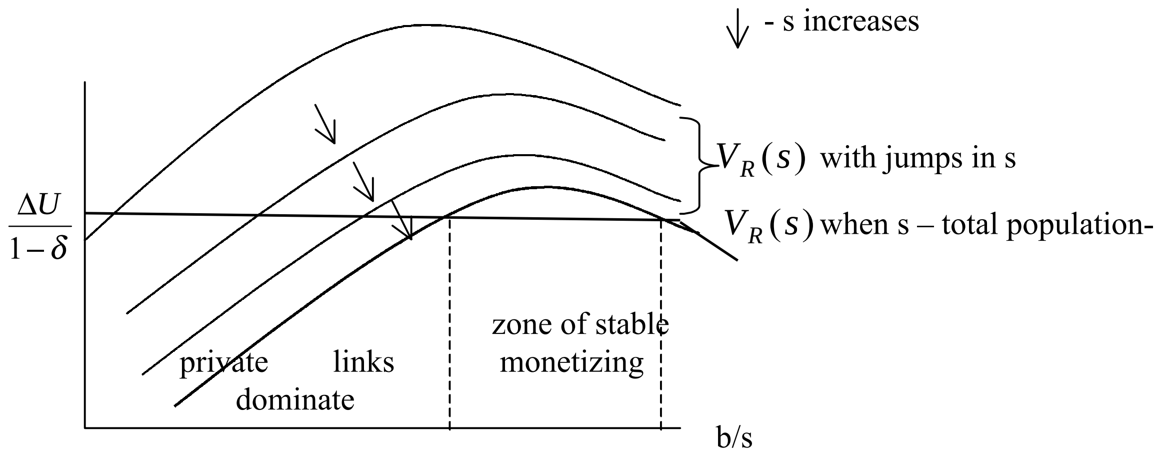
And for one having current barter partner:

$$V_R(s) = \Delta U + (\tfrac{s-b}{s})^2\delta V_R(s) + \tfrac{b}{s}\delta V(b) + \tfrac{b}{s}\cdot\tfrac{s-b}{s}\left[x\delta V_R(s) + (1-x)\delta V(s)\right]$$

The first term represents consumption with digestion cost. Barter lasts when both partners have not met buyers. An agent becomes a buyer when he met another buyer and transacted with him. An agent becomes a "market" seller when he has not met a buyer, but his former partner has. This case an agent can as well form new barter link. For simplicity we do not allow for the exogenous fission of the barter in this section.

$V_R(s)$ is increasing in $\frac{b}{s}$ approaching optimal ratio and then again decreasing. We do not consider here this decreasing region, in our terms it corresponds to the case where b is too great with respect to s. We will turn to the last issue in the discussion.

The inequality $\frac{\Delta U}{1-\delta} < V_R(s)$ ensures that in the "region of stable monetizing" lasting "forever" barter link worth less than the value of barter with possible breaching, so agents do not lock in barter.



Section II. Divisible Money.

II.1. Environment and Trade Protocol.

In the preceding sections we considered barter in the framework with indivisible money. The major reasons for that was simplicity of the analysis, or rather unmanageability of that in the setting with endogenous price. But we should point out that in the modern economic literature there appeared some papers see [2-4] that provide support for the analysis with indivisible money framework. Precisely *Green and Zhou*, *(1998)* [2] considered a version of *Kiyotaki and Wright* model in which agents can hold arbitrary amount of fiat money. *Green and Zhou* showed that with a seller-post protocol a continuum of stationary equilibria exists where all trade occur at a single price. Money holdings are treated as private information. Prices are set strategically. Under self-fulfilling expectations all sellers bid a single price and all buyers (with any money holdings) accept this price, and that is maximum price that he will accept. *Green and Zhou's* result is a bit suspicious because of assumptions about protocol they make. Agent's strategy does not depend on their money holdings, but rather on some exogenous reasons. People with huge money can act as sellers while clearly they can strictly benefit from long consumption.

In this section we develop a simple model that can act as a support for single-price equilibrium.

We work under the following assumptions:

Assumption 1. We consider only pairwise interactions. In each trading couple one part knows what is produced by the counterpart.

Assumption 2. There is no digestion cost. Good obtained via barter is of the same quality as the good bought at the monetary market.

So far the model coincides with the model with pairwise interactions.

Note that only one part in each trade group knows exactly what is produced by the counterpart, so the agents can only guess about what is needed for the barter exchange. So any pair form barter link with probability x.

Assumption 3. Stationarity of population not engaged in barter lock-ins at the moment. Thus two competing processes of barter formation and exogenous barter fission compensate each other.

Assumption 4. Money cumulative distribution function $F(m)$ and money probability distribution function $f(m)$ are common knowledge, while money holdings of each agent is his private information.

Assumption 5. Interactions and trading protocol.

1. b sellers from s are randomly selected to serve buyers.

2. s-b sellers are split into "nomads" and "settlers", nomads will visit settlers.
3. buyers or nomad sellers travel to sellers they are assigned to.
4. buyers or nomad agents offer their good for inspection for barter and make take-it-or-leave-offer: barter or money.
5. settlers in turn accept barter or not. If not $\rightarrow$ offer the price.
6. buyers or nomad sellers either accept the price $\rightarrow$ transact $\rightarrow$stop, or do not accept $\rightarrow$stop.

We will further define trader strategies as sellers optimal offer function and buyers reservation-price function.

Remember that in this setting it is always weakly optimal to accept barter, when the good offered is appropriate for consumption. Then under assumption 3 about population stationarity all the people with any money holdings engage in barter. Thus the distribution of money of the people not currently engaged in barter also is stationary. Because of discount it is always optimal to buy the good for money when barter it is not possible.

Definition. Agent's trading strategy is a pair of functions $\omega(m)$ that specifies the offer that he will make as a seller, when his current money holdings is m and $\rho(m)$ that specifies the maximum willingness to pay as a buyer, when his current money holdings is m.

Clearly a buyer with some money holdings accepts certain offers and rejects others. It is obvious that optimal decision will involve a cut-off level. Offers below it are accepted and offers above it are rejected. Note that as we are treating an agent as a representative one, offer functions and reservation price functions defined below depend not on his money holdings only.

A buyer is supposed to be able to pay his reservation price. We impose feasibility constraint $\rho(m) \leq m$.

Definition. Define the stationary distribution of offers as:

$\Omega(x) = F\{m \mid \omega(m) \leq x\}$

and the stationary distribution of reservation prices as:

$R(x) = F\{m \mid \rho(m) \leq x\}$,

where $F(m)$ is money cumulative distribution function.

To start with, a supposition will be made that all trades occur at a single price *p* (assuming that all buyers are going to buy at this price and all sellers are ready to sell). Then the stationary measure on trader money holdings under these assumptions will be characterized. Then a pair of offer strategy and reservation price constituting single-price equilibrium (in keeping to Buyesian-Nash concept) will be presented.

II.2. Stationary Money Allocation.

Let us suppose that there exists some cut-off value *p* that will act as a single price.

Lemma. In any single-price stationary equilibrium single price p=$\overline{m}/2$, where $\overline{m}$ is the upper boundary of money distribution. (the highest possible money holdings is $\overline{m}$)

Proof. Suppose $\overline{m}$ >2p. Then there are agents with money holdings $\eta > 2p$ that can consume in two subsequent periods. Note that postponing consumption when it is affordable is not optimal. So these agents will appear next period in the region $p < \eta < 2p$, but nobody will get to the region $\eta > 2p$, because people with $p < \eta < 2p$ are themselves consumers. So the initial distribution with $\overline{m}$ >2p was not stationary.

Suppose now $\overline{m}$ <2p. An agent with money holdings arbitrarily close but less than *p,* acts as a seller and obtains one *p* more. As a result his money holdings next period will be arbitrarily close to *2p.* So the initial distribution with $\overline{m}$ <2p was not stationary either. Q.E.D.

(A seller weakly prefers interacting with a buyer than with another seller. When an agent is not sure about getting the good himself he is willing to wait and serve a buyer.) Examine now the candidates for the stationary money distribution functions.

Each moment there are *F(p)* sellers and *1-F(p)* buyers at the market. Population currently at the market is normalized to one. Then:

$$\frac{\partial f(m \mid m < p)}{\partial t} = (1-x)\cdot f(m+p) - (1-x)\cdot f(m)\frac{1-F(p)}{F(p)} \quad \text{for } m \in (0, p)$$

The first term represents the inflow of former buyers that bought their good for money last period. The second - the outflow of sellers that sold their good for money last period. We include in the equation only the processes when money "changes the host". Clearly we should add the agents, terminated their barter links but subtract equal (because of assumptions 2 and 3) outflow of people to barter links.

$$\frac{\partial f(m \mid m \geq p)}{\partial t} = -(1-x)\cdot f(m) + (1-x)\cdot f(m-p)\frac{1-F(p)}{F(p)} \quad \text{for } m \in [p, 2p)$$

$$\frac{\partial f(m \mid m \geq 2p)}{\partial t} = 0 \quad \text{for } m \in [2p, \infty)$$

Setting time derivatives equal to zero we obtain the following functional equation for money distribution function

$$f(m+p) = f(m)\frac{1-F(p)}{F(p)} \quad \text{for } m \in [0, 2p)\ . \tag{6}$$

Note that any symmetric p.d.f. satisfy this equation. Note also that the set of solutions of functional equation

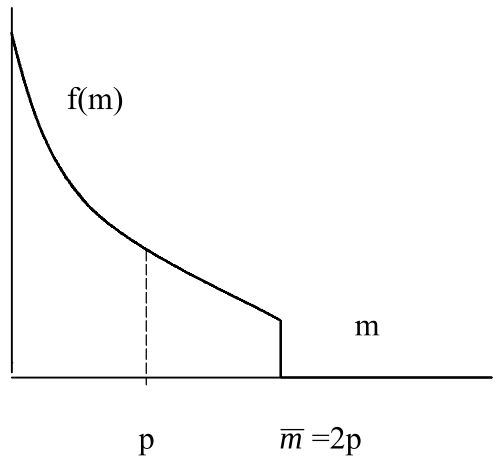
$$f(m+p)=\alpha \cdot f(m), \qquad (7)$$

with an arbitrary constant α is within the set of solutions of (6).

We cannot claim that we found the full set of solutions of (6), but all symmetric p.d.f.s are the solutions of (7), as well as some asymmetric: set: $f(m)=C\cdot\alpha^{m/b}$, (C,b arbitrary constants) for example. Moreover, functional equation (7) is linear in the sense that two solutions from two different sets: symmetric and asymmetric also form a solution of (7) and hence of (6).

We are interested mostly on positive $\alpha<1$ and positive C,b.

Corresponding probability distribution function of money is depicted on the figure:



II.3. Optimal Offer and Reservation Price

Note that with single-price value function assigned with money holdings is step-function. We denote $V(m \mid m<p)=V_s$, $V(m \mid m\geq p)=V_b$.

An agent values his money as the "predictor" of the future consumption path. Thus a seller making an offer o expects his future to be:

$$V(m)=xV_R(m)+(1-x)[(1-R(o))\delta V(m+o)+R(o)\delta V(m)] \qquad (8)$$

where $R(o)$ is the reservation price of the agent visited him. $1-R(o)$ is the probability that the offer *o* will be accepted. When an agent with reserve price *r* receives offers he forecasts the future as:

$$V(m) = xV_R(m) + (1-x)\left[\int_o^r (U + \delta V(m-q))d\Omega(q) + (1-\Omega(r))\delta V(m)\right]$$

where $\Omega(q)$ is the probability to receive an offer *q* from seller.

$V_R(m) = \alpha + \beta \cdot V(m)$, where $\alpha = \frac{U}{1-\delta P^2}, \beta = \frac{\delta(1-P^2)}{1-\delta P^2}$

In the seller-price-posting protocol for any possible offer *o* the buyer must issue either an acceptance or a rejection. The criterion for making this decision optimally is to accept *o* if and only if the offer is below buyer's full valuation of the good:

$$\forall o \quad \rho(m) \geq o \Leftrightarrow U + V(m-o) \geq V(m) \qquad (9)$$

The presumed optimal strategy in single-price equilibrium is that all agents currently not having at least *p* act as sellers. They are always willing to sell at *p*. Every agent having enough money is willing to purchase at *p*. This means that:

$\Omega(p) = 1$ and $\forall x < p \quad \Omega(x) = 0$

$R(p) = F(p)$ - if agent cannot pay, he acts as seller. Then the above equations (8) should include the probabilities of transition from seller to buyer and reciprocal states:

$$V_s = xV_R(s) + (1-x)\left[\frac{1-F(p)}{F(p)}\delta V_b + \frac{2F(p)-1}{F(p)}\delta V_s\right]$$

$$V_b = xV_R(b) + (1-x)(U + \delta V_s) \qquad (10)$$

$V_R(b) = \alpha + \beta \cdot V(b)$,

$V_R(s) = \alpha + \beta \cdot V(s)$, and α, β as above.

This system is similar to (5).

We use short notations showing that the function of money holdings is step-function one.

The system (10) is linear with respect to V_s, V_b. These V_s, V_b are functions of $F(p)$ and should satisfy the reduced form of the equation for optimal offer (9):

$$U + V_s = V_b \tag{11}$$

Equation (11) determines the value of $F(p)$ - the value of the constant in equation (7). Then the solution of equation (7) is the money distribution function, which is stationary in single-price equilibrium and consistent with the optimal strategy of "buyers". Thus all agents holding at least p will pay this money for the good. An equilibrium response for the sellers will be always to offer a price p. Deviating and offering higher price is not optimal as buyers can for sure buy from other sellers at price p (this works when discount is not significant). Seller does not benefit from obtaining slightly more than p, because this will lead to the consumption of the same indivisible good, but can "frighten away" a buyer.

As in all search-equilibrium models without production cost, there is a non-monetary equilibrium in this model: each agent simply gives his good away for free to anyone claiming needing it. This equilibrium has a greater amount of trade and hence a higher lever of welfare than does any monetary equilibrium. We do not believe that introduction of production cost removes this non-monetary equilibrium from the equilibrium set. We would rather note that the obvious difficulty with introduction of this non-monetary equilibrium is with assigning of the roles. When the good is given for free, everybody will be willing to be "buyers" and nobody will act as a producer. Then naturally the economy will improve the efficiency bidding up the price and establishing the "rules of fair play". Money as an object, which is infinitely costly to fabricate, may act as a confirmation of the fact that an agent is obeying the rules. Its presence in a transaction confirms that the one willing to buy today has "served his turn" as a seller. The stationarity of equilibrium in our economy, where people behave as assigned by their money holdings, ensures fulfilling of agents' expectations. The prevalence of single price results from self-fulfilling believes of agents.

Thus we presented a set {price, money c.d.f., sellers offer, buyers reservation price} which constitutes a Buyesian-Nash equilibrium in the described economy where all transactions occur at a single price.

We should confess that this very preliminary analysis heavily depends on some of the assumptions. The least "pleasant" of these is the assumption about trading protocol.

Another disadvantage of the analysis is that we cannot describe the behavior of the system out of stationary equilibrium, and even ensure convergence to any stationary parameter set. Nevertheless the system is stable with respect to small deviations from equilibrium. We can ensure the uniqueness of the price in equilibrium with given money c.d.f. within the set of single price equilibria, but there may be stationary c.d.f.s consistent with a dispersed price.

Tacking into account all these imperfections we believe that this model provides some support for conclusions drawn from the models with indivisible money. We should note that assumption 2 is not crucial and the extension of the results is possible also on the model with digestion costs.

Section III. Chain Effects in Monetizing.

Searching for possibility of these effects was stimulated by *Green and Zhou, (1998)* [5]. Not turning again to the critics about this paper we will note that their result is the single price equilibrium existence with the price $M = p\frac{1-m}{m}$. Here *p* is the price in question (nominal), *M* – total money stock, *m*- parameter of “unevenness with respect to money holdings”, namely the probability that randomly picked agent will be able to pay the price *p*.

Then in this setting “richer” agents impose an externality on the “poorer” pushing them out from monetary market and making them using barter as the only option available. This gives the following intuition about the monetary emission policy.

Note that “slightly” increasing *m* via monetary injection we can substantially decrease the price and increase the share of people “involved” in monetary market. This gives us the precise description of addressees of emission – it should be directed to the agents who are holding slightly less money than *p*.

We cannot observe the effects like this in our model with divisible money. Again note the difference: in *Green and Zhou* in contrast to the model presented in this paper agents behavior is determined by some exogenous reasons and may not be consistent with their money holdings.

Further the possibility of chain effects in microeconomic model based on the auction theory is discussed. Treating transactions as a series of auctions may appear to be not a bad description for "thin" markets. Asymmetry in bargaining power is also crucial for the analysis. The auctioneer may be treated as a monopoly, producing a good valuable for many consumers. Buyers might be treated as producers of goods valuable only for the monopoly.

The Formal Model.

Agents are competing in buying a single unit of good from seller A.

The good brings utility *U* for any agent. Their objective is to buy the good at the lowest lump-sum price. The indivisibility is crucial for the following analysis, but the good could be treated as tender or forward for the future product.

Agents expected utility takes the form:

$U \cdot y_i(m_1, m_2 ... m_n) + t_i(m_1, m_2 ... m_n)$

where $y_i(\cdot)$ is the probability of getting the good and $t_i(\cdot)$ is the expected transfer. m_i - agents money holdings.

We admit that introducing money in utility function is common in auction theory, but making this while discussing monetizing is not very natural. Monetizing may lead to considerable changes in economic parameters, for example, price increase. Then people will clearly become not indifferent between spending their money prior or after the monetizing takes place. Natural way of thinking about this in the auctions framework is to consider repeated auctions - the theory, which is at the moment very preliminary as well as complicated. We just state that people want the good today and want to purchase it at the least possible price, forecasting the future on their today experience and then making money in utility function have linear form.

Agents differ in their money holdings m_i (just falling from heaven). Ex interim the agent knows his money endowment, but can only guess about the money in the pockets of the neighbors. m_i of each agent is independently drawn from the set *M* and is distributed with c.d.f. $F(m)$, which is common knowledge.

Agents can produce an indivisible unit of something valuable for the seller and try to barter. They all produce goods that bring utility *V* for seller A (if brings at all), and this *V* is common knowledge. Agents can only guess about the exact preferences of the seller A. The probability that the good produced will be appropriate for barter is *x*, which is also common knowledge.

The auctioneer wants to extract as much money from the people as possible, but may accept barter as well. He establishes the following scheme for selling the good: two sequentialuent auctions. At the first the good can be bought only for money. This auction is English (first-price) with reserve price. If the good is not sold at the first auction it could be bartered at the second. At the barter auction the participants can make their offers (show the good they produced to the auctioneer). The auctioneer simply randomly chooses among the appropriate offers. The agents are allowed to make only take-it-or-leave-it offers. Thus they can participate either at the money, or at the barter auction. Agent is allowed to enter the auction room or to leave it freely (before the auction started), but any cheap talk is prohibited.

We should note that the described scheme is not the optimal mechanism designed by the auctioneer. He should rather allow all to participate at the first and then at the second auctions, allowing using bundled bids (money + barter). Although the scheme in the consideration is not optimal from the mechanism design point of view, it is suggestive one and depicts some special trade-off between money and barter described later. So we take the scheme as exogenously given.

Each agents observing how many competitors are present at the first auction room can calculate the optimal reserve price that will be set by auctioneer from:

$$n\int_{p}^{U}(sF©(s)+F(s)-1)F^{(n-1)}ds+F^{n}(p)(N-n)xV \xrightarrow[P]{} \max$$

n and N - the number of agents at the first auction and the total number of agents

For the optimal price: $p_n^* = V(N-n)x + \dfrac{1-F(p_n^*)}{f(p_n^*)}$ (*)

Agents with ex interim money holdings m_i decides where to participate comparing the probability of getting the good. The probability of winning at the first stage for agent "i" is $F^{n-1}(m_i)$ (the probability that he is the richest at the first auction).

At the barter stage the probability of winning is the product of: {Prob. that the good was not sold at the first stage}*{Prob. that the good produced is appropriate for barter}* {Prob. that she will be chosen as barter partner}. The first probability is clearly $F^{n-1}(p_{n-1}^*)$. The product of the remaining two is:

$$\sum_{m=0}^{n}\binom{N-n}{m}x^{m+1}(1-x)^{N-n-m}\frac{1}{m+1}=\frac{1}{N-n+1}\sum_{m=0}^{N-n}\binom{N-n+1}{m+1}x^{m+1}(1-x)^{N-n-m}=\frac{1-(1-x)^{N-n+1}}{N-n+1}$$

For the "marginal" agent holds almost with equality

$$F^{n-1}(m_i)=F^{n-1}(p_{n-1}^*)\frac{1-(1-x)^{N-n+1}}{N-n+1} \tag{12}$$

with p_n^* from the equation (*) for the optimal reserve price above.

Participation constraint $p_n^* < m_i < p_{n-1}^*$ should be added.

Note that F^n (m) is monotonous function of money holdings

All the agents with ex interim money higher than $m_j^*(N)$- the solution of the "marginal" constraint will show up at the money auction, the others are using barter.

Note that right hand side of (12) is the product of two independent probabilities:

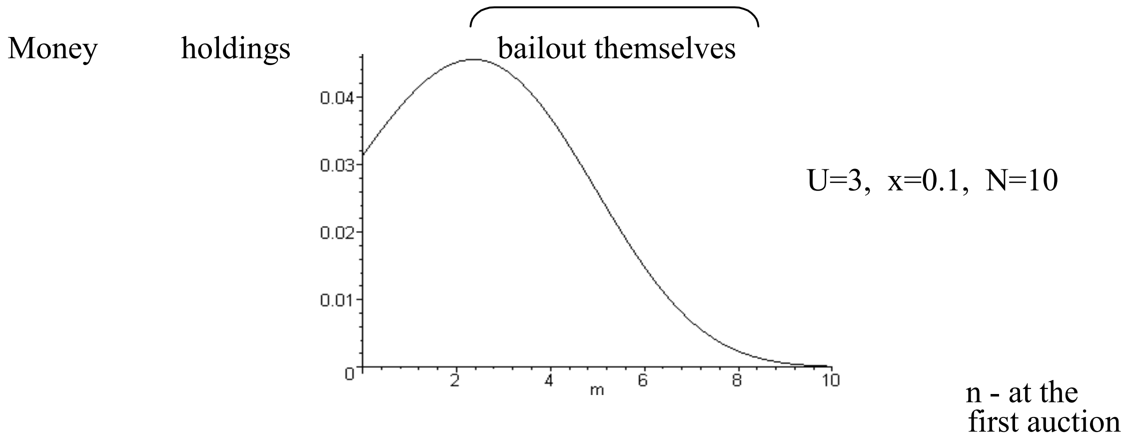
$\Pi(n)=\dfrac{1-(1-x)^{N-n+1}}{N-n+1}>0$ and $F^{n-1}(p_{n-1}^*)>0$ which is monotonous in p .

$\dfrac{d\Pi}{dn}>0$, while p_n^* could be either decreasing or increasing in n depending on the "flatness" of the distribution.

i.e. iff $f^2(f'+1)>f'(1-F)$ then $\dfrac{dp_n^*}{dn}<0$ (uniform distribution satisfies the condition needed)

The "marginal" agent choosing the money auction could affect the “attractiveness” of the barter auction for the next ranked by money holdings. Namely, when this «marginal» agent obtains enough money he is persuaded to take part at the monetary auction. $\Rightarrow$ Doing so he decreases the outside option for the auctioneer. $\Rightarrow$ That decreases the reserve price set by the auctioneer $\Rightarrow$ That decreases the likelihood that the good will be traded at the barter auction. $\Rightarrow$ That makes the barter auction less attractive for the agent ranked next in money holdings. $\Rightarrow$ He may also choose to participate in money auction.

The above is true for the cases of f flat enough, when the dependence $F^{n-1}(p^*_{n-1})\frac{1-(1-x)^{N-n+1}}{N-n+1}$ on n has a pick.



Then the loci of stable equilibria (people stacked in barter) - the upward sloping region of the curve. Here the agent moving from barter auction increases the "barrier of entry" for the next ranked in money holdings.

The downward sloping area is unstable - people could be bailout from using barter by the described scheme. There is an effect similar to chain reaction: giving money to the right (marginal) person we can substantially increase the number of people using the money. Note that in general the situation when the good is sold at the monetary auction is more efficient: the good is sold for sure. At the barter stage it may not be sold at all. The reason for using barter is the difference between money and consumption goods that we seem managed to capture by this scheme. People are *different in money holdings* and this difference is either perpetual or cannot be overcame at the moment, while people are *equally able to produce* something valuable. Then they prefer to stack in inefficient equilibrium where they may be very unhappy *but equally unhappy*. The reason for barter existence could also betraced to price discrimination idea presented in *Guriev, Kvasov* [9].

Section IV. Discussion and Policy Implications.

We discussed the trade-off between barter and market exchange as a trade-off between two types of information. Bilateral barter link corresponds to private information, whereas market trade corresponds to network system. Distributing money then we endow people not only with "pieces of paper" which they may use in their transactions, but also with an access to informational network that we call monetary market.

In this section we discuss the results of all the models presented in the paper. We will start with some general remarks. Firstly, in all the models money appeared to be neutral in the sense that lump-sum (equal to everybody alive) transfer does not affect the real variables. Secondly, money always improves welfare, when properly distributed. Proper distribution in all discussed models with indivisible money means that monetary injection should be directed to the "poor" agents currently searching for a barter partner. That is turning some "market" sellers into "market" buyers. The question how much money to distribute is beyond the scope of the models described, but naturally the eventual number of people with money should not exceed the number of people without money.

What will this monetary injection make to barter?

We discussed two types of barter: 1) voluntary when people barter for the goods that they would anyway buy from the same people and 2) inferior barter involving digestion costs which can be traced to the various types of transaction costs, re-barter, etc. As we treat barter as an information about the partner, voluntary barter implies knowing the right person to transact with. This situation is efficient one and could not be improved. We may cite Wallace theorem: money is accepted only when it makes better allocation achievable.

For voluntary barter, which is estimated, (*see Aukutsionek* [1]) for 60% of total barter size, the policy suggestion: do not touch it.

Inferior barter is not an efficient one, thus welfare can be improved by monetizing. Our analysis of this type of barter suggests the existence of two adjacent equilibria zones. In one of this zones private links dominate the network market structure. The monetary injection should be sufficient to "trigger" the economy into a stable monetizing zone (see the figure and a discussion above).

The policy suggestion for inferior barter is a big monetary injection to the sellers.

We considered only the economies with the number of buyers lower than the number of sellers. The opposite case is rather unnatural with indivisible money, and in the model with divisible money this implies only that the price was underestimated. Justifying enough is relation the case of too many buyers with inflation, as people cannot buy the good for one unit of money because of excess demand but probably will buy for two. Cases of inflation driven barter were studied in some papers with random-matching models, see *Banerjee, Maskin* [2]. But the random-matching framework has proven to be rather clumsy for discussing inflation cases. Treatment of barter as a second currency could be promising enough and the abundant literature on currency substitution may be of a great value there. See for example *Uribe* [17], discussing cash-in-advance model, where the effects connected with accumulating experience in using second currency (or barter if this substitution is allowed) are examined.

Thus monetary injection should not be too big. Otherwise it may trigger inflation driven barter. The microeconomic story with two sequentialuent auctions and *Green and Zhou* [5] suggest that addressees of the injection matter not only for avoiding of inflation expectations, but also because of possibility of chain effects in monetizing.

The policy suggestion then is to distribute money transfers between some "middle-men" that have money holdings just being about entering market. These agents may be detected statistically as the people who use barter only occasionally.

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Appendix.

Nomad agent chosen the production place faces the probability to be selected as a reciprocal exchange partner:

$$S_t^R = \frac{(1-\mu)s}{L} \sum_{n=0}^{x\mu s-1} \sum_{m=0}^{n} \binom{x\mu s - 1}{n} \cdot \left(\frac{1}{xL}\right)^n \left(1 - \frac{1}{xL}\right)^{x\mu s - 1 - n} \binom{n}{m} x^{m+1} (1-x)^{n-m} \frac{1}{m+1}$$

This formulae is the product of independent probabilities 1) that the place with the seller present was chosen, 2) *n* of other «nomads» chosen the same place, 3) *m* from these «nomads» have the appropriate preferences for mutually beneficial barter, 4) the given «nomad» was selected from the group of these «nomads» with appropriate preferences. This formulae as well accounts for the fact that «nomad» herself is appropriate for barter with the seller.

First lets calculate the sum over *m*:

$$\sum_{m=0}^{n} \binom{n}{m} x^{m+1} (1-x)^{n-m} \frac{1}{m+1} = \frac{1}{n+1} \sum_{m=0}^{n} \binom{n+1}{m+1} x^{m+1} (1-x)^{n-m} = \frac{1-(1-x)^{n+1}}{n+1}$$

the sum over *n* could be calculated as the difference of two components. The first:

$$\frac{(1-\mu)s}{L} \sum_{n=0}^{x\mu s-1} \binom{x\mu s - 1}{n} \cdot \left(\frac{1}{xL}\right)^n \left(1 - \frac{1}{xL}\right)^{x\mu s - 1 - n} \frac{1}{n+1} =$$

$$\frac{(1-\mu)s}{L} \frac{1}{x\mu s} \sum_{n=0}^{x\mu s-1} \binom{x\mu s}{n+1} \cdot \left(\frac{1}{xL}\right)^{n+1} xL \cdot \left(1 - \frac{1}{xL}\right)^{x\mu s - 1 - n} =$$

$$= \frac{(1-\mu)s}{\mu s} \left(1 - \left(1 - \frac{1}{xL}\right)^{x\mu s}\right) \xrightarrow[L\to\infty]{} \frac{1-\mu}{\mu} \left(1 - e^{-\mu s}\right)$$

the second:

$$\frac{(1-\mu)s}{L}\sum_{n=0}^{x\mu s-1}\binom{x\mu s-1}{n}\cdot\left(\frac{1}{xL}\right)^{n}\left(1-\frac{1}{xL}\right)^{x\mu s-1-n}\frac{(1-x)^{n+1}}{n+1} =$$

$$\frac{(1-\mu)s}{L}\frac{1}{\mu s}\sum_{n=0}^{x\mu s-1}\binom{x\mu s}{n+1}\cdot\left(\frac{1-x}{xL}\right)^{n+1} xL\cdot\left(1-\frac{1}{xL}\right)^{x\mu s-1-n} =$$

$$=\frac{1-\mu}{\mu}\left(\left(1-\frac{1}{L}\right)^{x\mu s}-\left(1-\frac{1}{xL}\right)^{x\mu s}\right)\xrightarrow[L\to\infty]{}\frac{1-\mu}{\mu}\left(e^{-x\mu s}-e^{-\mu s}\right)$$

Thus the probability for a «nomad» to be selected as a reciprocal exchange partner

s: $B_u^R=\frac{1-\mu}{\mu}\left(1-e^{-x\mu s}\right)$